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A Growth and Yield Model for Thinned Stands of Yellow-Poplar

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A Growth and Yield Model for Thinned Stands of Yellow-Poplar

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ABSTRACT. Simultaneous growth and yield equations were developed for predicting basal area growth and cubic-foot volume growth and yield in thinned stands of yellow-poplar. A joint loss function involving both volume and basal area was used to estimate the coefficients in the system of equations. The estimates obtained were analytically compatible, invariant for projection length, and numerically equivalent with alternative applications of the equations. Given estimates of basal area and cubic-foot volume from these equations, board-foot volumes can also be calculated.

As an adjunct to the stand-level equations, compatible stand tables were derived by solving for the parameters of the Weibull distribution from attributes predicted with the stand-level equations. This procedure for estimating the parameters of the diameter distributions of the stands before thinning gave reasonable estimates of number of trees, basal area, and cubic-foot volume per acre by diameter class. The thinning algorithm removes a proportion of the basal area from each diameter class and produces stand and stock tables after thinning from below that are consistent with those generated before thinning.

ADDITIONAL KEY WORDS. Liriodendron tulipifera, mensuration, thinning, modeling.

INTRODUCTION

IN THE EASTERN UNITED STATES, yellow-poplar (*Liriodendron tulipifera* L.) is an important commercial species that is cut primarily for lumber and veneer. Because tree size and quality greatly influence yields of these products, thinning is an important silvicultural tool in yellow-poplar management. Most stands of yellow-poplar can produce a number of lumber- and veneer-size trees without thinning; however, thinning concentrates growth on the best and largest trees. Reliable estimates of stand growth and yield are needed to determine optimal thinning regimes.

Beck and Della-Bianca (1972) published equations for predicting basal area growth and cubic-foot volume growth and yield in yellow-poplar stands thinned to various levels of basal area. However, flexible models that supply information about the diameter distributions—and hence product distributions—are needed to better evaluate the effects and results of various thinning options.

The objectives of this study were to develop a growth and yield model for yellow-poplar that can be used to evaluate thinning options. This model should be efficient to use and provide detailed information about stand structure. To accomplish these objectives, we

1. Developed a stand-level model for thinned stands of yellow-poplar, and
2. Derived diameter distributions from predicted stand attributes.

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LITERATURE REVIEW

Stand-Level Models

The first yield predictions in the United States were made using normal yield tables for natural even-aged stands of a given species. Temporary plots in stands of "normal" stocking were used to construct these tables through graphical techniques. Volume and yield tables of this type for yellow-poplar in the southern Appalachians were presented by McCarthy (1933).

MacKinney and others (1937) suggested the use of multiple regression to construct variable-density yield equations. Subsequently, MacKinney and Chaiken (1939) used a multiple regression analysis to construct a yield prediction equation for loblolly pine stands. Since that time, many investigators have used multiple regression to construct stand aggregate growth and/or yield expressions (Schumacher and Coile 1960; Coile and Schumacher 1964; Goebel and Warner 1969; Burkhart and others 1972a, 1972b; and others).

Until the early 1960's, independent equations were developed to predict growth and yield, often resulting in inconsistent and illogical results. Buckman (1962) introduced a model for red pine where yield was obtained through mathematical integration of the growth equation over time, thus taking into account the logical relationship which should exist between growth and yield equations. Clutter (1963) discussed this concept of compatibility between growth and yield prediction in detail and developed a compatible growth and yield model for natural loblolly pine stands.

Sullivan and Clutter (1972) refined Cutter's equations to develop a simultaneous growth and yield model for loblolly pine that provided not only analytically, but also numerically consistent growth and yield predictions. This growth and yield model has been successfully used for loblolly pine (Brender and Clutter 1970, Sullivan and Williston 1977, Murphy and Stemitzke 1979, Burkhart and Sprinz 1984), shortleafpine (Murphy and Beltz 1981), slash pine (Bennett 1970), and yellow-poplar (Beck and Della-Bianca 1972).

Diameter Distribution Models

Stand yields have also been predicted using diameter distribution analysis procedures. In such cases it is often assumed that the underlying diameter distribution of the stand can be adequately characterized by a probability density function (pdf).

Clutter and Bennett (1965) fitted the beta distribution to observed diameter frequency data from old-field slash pine plantations, and, from this, developed variable density stand tables. Bennett and Clutter (1968) used these stand tables to estimate multiple-product yields for slash pine plantations. The parameters of the beta distribution that approximated the diameter distribution were predicted from stand variables (age, site index, and density). The number of trees and volume per acre in each diameter class were then calculated, and per acre yield estimates were obtained by summing over the diameter classes of interest.

Following these same procedures, McGee and Della-Bianca (1967) successfully fitted the beta distribution to describe diameter distributions in even-aged natural stands of yellow-poplar. From this diameter distribution information. Beck and Della-Bianca (1970) then obtained yield estimates for even-aged stands of unthinned yellow-poplar. A similar approach was used for loblolly pine plantations by Lenhart and Clutter (1971), Lenhart (1972), and Burkhart and Strub (1974). In each of these cases, the minimum and maximum diameters defining the limits of the distributions, as well as the pdf parameters, were predicted from functions of stand characteristics.

The beta distribution is very flexible in shape and can approximate a wide range of diameter distributions. In addition, the pdf has finite limits which constrain all diameters to be within upper and lower bounds. A disadvantage of this distribution, however, is that the pdf must be numerically integrated to obtain probabilities over various ranges of the random variable, i.e., to obtain the proportion of trees in each diameter class, as the cumulative distribution function (cdf) does not exist in closed form.

More recently, the Weibull distribution has been widely applied for describing diameter distributions. The pdf is flexible in shape, the parameters are reasonably easy to estimate, and the cdf exists in closed form—a major advantage over the beta pdf. The Weibull pdf exists in either a two or three parameter form, the three parameter pdf having the advantage of increased flexibility.

First used as a diameter distribution model by Bailey (1972), the Weibull distribution has been applied to a wide range of situations. For example, it has been used to describe diameter distributions in loblolly pine plantations (Smalley and Bailey 1974a, Schreuder and Swank 1974, Feduccia and others 1979, Cao and others 1982, Amateis and others 1984), slash pine plantations (Dell and others 1979, Bailey and others 1982), shortleaf pine plantations (Smalley and Bailey 1974b), longleaf pine plantations (Lohrey and Bailey 1976), natural stands of loblolly pine (Burk and Burkhart 1984), and white pine (Schreuder and Swank 1974). Bailey and Dell (1973) concluded no other distribution proposed exhibited as many desirable features as the Weibull.

Given an appropriate density function, Strub and Burkhart (1975) presented a class-interval-free method for obtaining yield estimates over specified diameter class limits. The general equation form is given by

$$V = N\int_l^u g(D)f(D)\,dD$$

where

V = expected stand volume per unit area,
N = number of trees per unit area,
D = dbh,
$g(D)$ = individual tree volume equation,
$f(D)$ = pdf for D, and
l, u = lower and upper merchantability limits, respectively, for the product described by g(D).

Using attributes from a whole stand model and the relationship given by the class-interval-free equation presented by Strub and Burkhart (1975), Hyink (1980) introduced a method of solving for the parameters of a pdf approximating the diameter distribution. The approach was to predict stand average attributes of interest for a specified set of stand conditions, and use these estimates as a basis to "recover" the parameters of the underlying diameter distribution using the method of moments technique.

When constructed independently, even from the same data set, stand average and diameter distribution models, which give different levels of resolution, do not necessarily produce the same estimates of stand yield for a given set of stand conditions (Daniels and others 1979). The advantages of the procedure outlined by Hyink are ability to partition total yield by diameter class, mathematical compatibility between the whole stand and diameter distribution based yield models, and consistency among the various stand yield estimates.

Based on this procedure, Frazier (1981) developed a method to approximate the diameter distributions of unthinned plantations of loblolly pine from whole stand predictions of stand attributes using the beta and Weibull pdfs. Using the same concept, Matney and Sullivan (1982) developed a model for thinned and unthinned loblolly pine plantations. Cao and others (1982) used the Weibull function to derive diameter distributions from predicted stand attributes for thinned loblolly pine plantations. Cao and Burkhart (1984) used a similar approach with a segmented Weibull cumulative distribution to derive empirical diameter distributions from predicted stand attributes for thinned loblolly pine plantations. Hyink and Moser (1983) extended the idea and developed a generalized framework for projecting forest yield and stand structure using diameter distributions.

MODEL DEVELOPMENT

Several desirable properties were sought when deriving a growth and yield model for thinned stands of yellow-poplar. In particular, we wanted the equations to exhibit analytic compatibility between growth and yield, invariance for projection length, and numeric equivalency between alternative applications of the equations. In addition to whole stand volume and basal area, we also wanted to derive stand tables to provide flexibility for evaluating the full range of utilization options. Consequently, another goal was to derive stand tables that are compatible with the whole stand values.

The model for thinned stands of yellow-poplar was developed in two stages. In the first stage, equations to predict stand-level attributes were obtained. In the second stage, stand tables were derived from the whole-stand attributes by solving for parameters in a theoretical diameter distribution model (in this case the Weibull distribution was used) while ensuring compatibility between the whole stand and diameter distribution estimates of the stand-level attributes.

Plot Data

Data for this study were collected by the U.S. Forest Service, Southeastern Forest Experiment Station, from 141 circular, ¼-acre plots established in the Appalachian Mountains of North Carolina (93 plots), Virginia (31 plots), and Georgia (17 plots). The plots contained 75 percent or more yellow-poplar in the overstory, were free from insect and disease damage, and showed no evidence of past cutting (Beck and Della-Bianca 1972).

Each plot was thinned (using low thinning) at the time of installation to obtain a range of basal areas for different site-age combinations. Site index at age 50 was determined for each plot with an equation published by Beck (1962). Volumes and basal areas were computed when the plots were thinned and again after five growing seasons. At the time of initial plot establishment, the stands ranged from 17 to 76 years in age, 74 to 138 feet in site index (base age 50 years), and 44 to 209 sq ft per acre in basal area.

Table 1 shows a summary of the plot data before and after the first thinning (measure 1), before and after the second thinning (measure 2), 5 years after the second thinning (measure 3), and 10 years after the second thinning (measure 4). Basal area and cubic-foot volume growth between the four measurement periods are presented in Table 2.

Stand-Level Component

When fitting the stand-level components, we used the models of Beck and Della-Bianca (1972) as a starting point because these models exhibit desirable properties and they were successfully fitted to the first 5-year growth data from the yellow-poplar plots. Beck and Della-Bianca fitted the following models (adapted from Sullivan and Clutter 1972) for prediction of basal area and cubic volume at some projected age when site index, initial age, and basal area are given:

$$\ln(Y_2) = b_0 + b_1(S^{-1}) + b_2(A_2^{-1}) + b_3(A_1/A_2)\ln(B_1) + b_4(1-A_1/A_2) + b_5(S)(1-A_1/A_2) \qquad (1)$$

where

Y = stand volume per unit area at some projected age, A_2
S = site index,
B_1 = present basal area per unit area, and
A_1 = present age.

When $A_2 = A_1 = A$ and $B_2 = B_1 = B$, equation (1) reduces to the general yield model

$$\ln(Y) = b_0 + b_1(S^{-1}) + b_2(A^{-1}) + b_3\ln(B) \quad (2)$$

The yield prediction model (1) was derived by substituting a basal area projection equation for the basal area term in the general yield model (2). Therefore, inserting $\ln(Y_2;)$, A_2, and $\ln(B_2)$ into equation (2) and setting the resulting expression equal to the right side of equation (1) and solving the equality for $\ln(B_2)$ gives the basal area projection model

$$\ln(B_2) = (A_1/A_2)\ln(B_1) + (b_4/b_3)(1-(A_1/A_2) + (b_5/b_3)(S)\ (1-A_1/A_2) \quad (3)$$

Beck and Della-Bianca (1972) used ordinary least squares to estimate the coefficients in (1) and substituted the ratios b_4/b_3 and b_5/b_3 as parameter estimates in the basal area projection equation (3) to ensure that exact numerical equivalency would result when projecting future volume from (1) and when projecting future basal area from (3) and solving for future volume by substitution of appropriate values into (2).

In our analyses, equation (1) was fitted by ordinary least squares to each of the growth periods and standard F-tests were performed to determine if separate coefficients were needed for each period or if data from some of the periods could be combined. From these tests, we determined that two sets of coefficients were needed—one for the growth period after one thinning and a second for the growth periods following two thinnings. The second thinning apparently altered stand structure and vigor so that growth relationships were significantly affected.

After determining that separate coefficients were needed for the growth periods following one thinning and following two thinnings, final estimates of the parameters in the volume and basal area projection equations were computed by using a simultaneous fitting procedure. This procedure, applied previously by Burkhart and Sprinz (1984) to data from thinned loblolly pine plantations, involves minimizing the loss function:

$$F = \frac{\sum_i (Y_i - \hat{Y}_i)^2}{\hat{\sigma}_Y^2} + \frac{\sum_i (B_i - \hat{B}_i)^2}{\hat{\sigma}_B^2} \quad (4)$$

where

Y_i and $\hat{Y}_i$ = observed and predicted volume values, respectively,
B_i and $\hat{B}_i$ = observed and predicted basal area values, respectively,
$\hat{\sigma}_Y^2$ and $\hat{\sigma}_B^2$ = estimates of the variance about the regression lines for volume and basal area, respectively, computed as the mean square error from ordinary least squares fits of equations (1) and (3).

TABLE 1. Yellow-poplar plot data summary.

Time of measure[a] and stand variable[b]		Number of plots	Minimum value	Mean value	Maximum value
Measure 1					
	Age Site Ntb Nta Ntr Bab Baa Bar Cvb Cva Cvr Bvb Bva Bvr	141	1.7741043e+32	46.9 107.8 231.8 105.1 126.7 134.8 85.4 49.5 5,772.2 3,857.8 1,881.0 18,671.9 14,418.2 4,253.6	7.6138432e+50
Measure 2					
	Age Site Ntb Nta Ntr Bab Baa Bar Cvb Cva Cvr Bvb Bva Bvr	141	2.2743228e+28	41.9 107.8 105.1 83.5 21.6 97.4 86.0 11.4 4,588.7 4,112.6 476.1 18,221.3 16,963.7 1,257.5	8.1138340e+47
Measure 3					
	Age Site Ntb Nta Ntr Bab Baa Bar Cvb Cva Cvr Bvb Bva Bvr	140	2.7742828e+31	57.1 107.7 81.6 81.6 0 97.6 97.6 0 4,889.9 4,889.9 0 21,455.9 21,455.9 0	8.6138256e+38

TABLE 1. Continued

Time of measure[a] and stand variable[b]		Number of plots	Minimum value	Mean value	Maximum value
Measure 4					
	Age	138	3.37428280e+31	62.4	9.11382482e+40
	Site			107.6	
	Ntb			80.7	
	Nta			80.7	
	Ntr			0	
	Bab			110.0	
	Baa			110.0	
	Bar			0	
	Cvb			5,621.3	
	Cva			5,621.3	
	Cvr			0	
	Bvb			25771.3	
	Bva			25,771.3	
	Bvr			0	

[a]Plot data before and after first thinning (measure 1), before and after second thinning (measure 2), 5 years after second thinning (measure 3), and 10 years after second thinning (measure 4).

[b]Age = age of stand (years).
Site = site index (feet, base age 50 years).
Ntb = number of trees/ac prior to thinning.
Nta = number of trees/ac after thinning.
Ntr = number of trees/ac removed in thinning.
Bab = basal area (sq ft/ac) prior to thinning.
Baa = basal area (sq ft/ac) after thinning.
Bar = basal area (sq ft/ac) removed in thinning.
Cvb = cubic-foot volume/ac prior to thinning.
Cva = cubic-foot volume/ac after thinning.
Cvr = cubic-foot volume/ac removed in thinning.
Bvb = board-foot volume/ac prior to thinning.
Bva = board-foot volume/ac after thinning.
Bvr = board-foot volume/ac removed in thinning.

Beginning with coefficients estimates from the ordinary least squares fit of (l), the coefficients of models (1) and (3) were adjusted through an iterative process until *F* in the loss function was minimized. This process of simultaneously fitting the two models (with the imposed restriction that the coefficients in the basal area equation are equal to the appropriate ratios of the volume equation coefficients) results in a system of equations that are compatible and numerically consistent. Different weights could be assigned to the two components, but we felt that for management decisions involving thinning equal weight should be given to both volume and basal area projection. The simultaneous estimation procedure is more statistically efficient (in that the basal area growth information is used in the fitting) and produces more stable estimates of the basal area equation coefficients for varying units of measure and merchantability standards in (1) than does the derivation of coefficients in (3) from the least squares fit of (l) (Burkhart and Sprinz 1984). The basal area and cubic-foot volume equations from the simultaneous fitting procedure and their related fit statistics are presented in Tables 3 and 4. In the evaluation process, current volume yield values (i.e., observations for which $A_2 = A_1 = A$) were used in addition to the growth data, thus doubling the number of yield observations. Due to the model structure, current basal area values could not be used.

Beck and Della-Bianca (1975) predicted the ratio of board-foot volume to basal area using dominant stand height and residual quadratic mean stand diameter. In this study, we developed the following equation from the plot data to relate board-foot volume to stand basal area and cubic-foot volume.

TABLE 2. Summary of basal area and cubic-foot volume growth during the 5-year periods between the four plot measurements.

Growth period	Variable[a]	Minimum value	Mean value	Maximum value	Mean annual growth
5 years	B1	25	85.4	153	
after first	B2	38	97.4	171	
thinning	Bg	5	12.0	33	2.4
	V1	1,106	3,857.8	8,102	
	V2	1,224	4,588.7	9,398	
	Vg	318	794.7	1,920	158.9
5 years	B1	22	86.0	150	
after second	B2	31	97.6	164	
thinning	Bg	4	12.5	32	2.5
	V1	722	4,112.6	8,109	
	V2	1,222	4,889.9	9,030	
	Vg	260	790.7	2,190	158.1
10 years	B1	31	97.6	164	
after second	B2	40	110.0	178	
thinning	Bg	-1	12.9	26	2.6
	V1	1,222	4,889.9	9,030	
	V2	1,565	5,621.3	10,070	
	Vg	-61	856.8	1,740	171.4

[a]B1 = basal area (sq ft/ac) at beginning of growth period.
B2 = basal area (sq ft/ac) at end of growth period.
Bg = B2 - Bl, i.e.. 5 years growth.
V1 = cubic-foot volume/ac at beginning of growth period.
V2 = cubic-foot volume/ac at end of growth period.
Vg = V2 - V1, i.e., 5 years growth.

TABLE 3. Simultaneous growth and yield equations[a] for prediction of total cubic-foot volume and basal area per acre.

$\ln(Y_2) = b_0 + b_1(S^{-1}) + b_2(A_2^{-1}) + b_3(A_1/A_2)\ln(B_1) + b_4(1 - A_1/A_2) + B_5(S)(1 - A_1/A_2)$ $\ln(B_2) = (A_1/A_2)\ln(B_1) + (b_4/b_3)(1 - A_1/A_2) + (b_5/b_3)(S)(1 - A_1/A_2)$	
For stand thinned once	For stands thinned twice
$b_0 = 5.35740$	$b_0 = 5.33115$
$b_1 = -102.45728$	$b_1 = -97.95286$
$b_2 = -21.95901$	$b_2 = -25.19324$
$b_3 = 0.97473$	$b_3 = 0.98858$
$b_4 = 4.11893$	$b_4 = 5.84476$
$b_5 = 0.01293$	$b_5 = 0.00018$

[a]Where

Y_2 = predicted total cubic-foot volume per acre at projected age, A_2.
A_1 = initial age.
S = site index, base age 50 years (feet).
B_1 = initial basal area per acre (sq ft).
B_2 = predicted basal area per acre (sq ft) at A_2
ln = natural (Naperian) logarithm.

TABLE 4. Fit statistics for evaluating cubic-foot volume and basal area prediction from the simultaneous growth and yield equations.

Equation	Number of observations	Minimum residual value[a]	Mean residual value	Mean absolute residual value	Maximum residual value	Standard deviation of residual values	R^{2b}
Cubic-foot volume	840	-808.91	6.68	156.46	1250.39	219.74	0.9865
Basal area	419	-13.66	0.78	2.9	16.62	3.69	0.986

[a]A residual value is the difference between the observed and predicted value of the dependent variable:

$r_i = Y_i - \hat{Y}_i$

[b]The R^2 value was computed as follows:

$$R^2 = 1 - \frac{\sum_{i=1}^{n} r_i^2}{\sum_{i=1}^{n} (Y_i - \overline{Y})^2}$$

where

Y_i	=	i^{th} observed value of the dependent variable.
$\hat{Y}_i$	=	i^{th} predicted value of the dependent variable.
$\bar{Y}$	=	mean value of the dependent variable.
r_i	=	i^{th} residual value as defined above in footnote a.
n	=	number of observations.

$$BFV = 1363.09165 - 306.96647(B) + 10.26187(CFV)$$
$$R^2 = 0.9730 \qquad s = 1785.1 \tag{5}$$

where

BFV	=	board-foot volume per acre, International 1/4-inch rule, for all trees in the 11-inch dbh class and above to an 8-inch top diameter (ob) (1-foot stump).
B	=	basal area per acre (sq ft) of all stems.
CFV	=	total cubic-foot volume per acre.
R^2	=	coefficient of determination.
s	=	root mean square error.

Given equations for estimating the total stand cubic volume and basal area, the board-foot volume of a selected portion of the stand according to an 8-inch top diameter outside bark can be estimated. This approach does not allow sufficient flexibility, however, to account for rapidly changing utilization standards. Thus an extremely valuable adjunct to the overall stand values is a stand table. When computing a stand table it is important that it be logically and consistently related to the overall stand characteristics.

Stand Table Generation

PARAMETER RECOVERY PROCEDURE

The parameter recovery procedure introduced by Hyink (1980) and further discussed and developed by Frazier (1981), Matney and Sullivan (1982), Cao and others (1982), Hyink and Moser (1983), and Cao and Burkhart (1984) was used to obtain estimates of the parameters of the Weibull pdf, which was used to describe the diameter distributions of yellow-poplar stands before and after thinning. The recovery method was selected because it provides compatible whole stand and diameter distribution estimates of specified stand attributes.

The Weibull pdf exists in either a two or three parameter form. These two forms are defined as follows. Three parameter Weibull density

$$f_z(z;a,b,c) = \begin{cases} \left(\frac{c}{b}\right)\left(\frac{z-a}{b}\right)^{c-1} \exp\left[-\left(\frac{z-a}{b}\right)^{c}\right] & a,b,c > 0 \\ & z > a \\ 0, otherwise. & \end{cases}$$

Two parameter Weibull density

$$f_x(x;b,c)=\begin{cases}\left(\frac{c}{b}\right)\left(\frac{x}{b}\right)^{c-1}\exp\left[-\left(\frac{x}{b}\right)^{c}\right] & y,b,c>0\\ 0, otherwise\end{cases}$$

where

a = the location parameter,
b = the scale parameter,
c = the shape parameter,
Z = the random variable (diameter), and
$X = Z - a$.

With the general diameter distribution yield function,

$$Y_i = N \int_l^u g_i(x) f(x;\, \underline{\theta})\, dx \qquad (6)$$

where

Y_i	=	total per unit area value of the stand attribute defined by $g_i(x)$
$g_i(x)$	=	stand attribute as a function of x
$f(x;\underline{\theta})$	=	pdf for x
N	=	number of trees per unit area
l, u	=	lower and upper diameter limits, respectively, for the product described by $g_i(x)$,

integration over the range of diameters, X, for any $g_i(x)$, gives the total per unit area value of the stand attribute defined by $g_i(x)$. Average diameter, basal area per acre, and total cubic volume per acre are examples of such stand attributes. The number of stand attribute equations must equal the number of parameters to be estimated in order to solve the system of equations for recovery of the pdf parameters.

Letting $g_i(x)$ equal x^i, one obtains the i[th] noncentral moment of X as

$$E(X_i) = \int_{-\infty}^{\infty} X^i f(x;\, \underline{\theta})\, dx$$

and the parameter recovery system is simply the method of moments technique of pdf parameter estimation (Mendenhall and Scheaffer 1973).

In the case of forest diameter distributions, the first noncentral moment, $E(X^2)$, is estimated by

$$\sum x_i \;/\; N = \bar{x},$$

the arithmetic mean diameter of the stand, and the second noncentral moment, $E(X^2)$, is estimated by

$$\sum x_i^2 \;/\; N = \overline{x^2} = \text{basal area/acre/0.005454N},$$

(the quadratic mean diameter of the stand) where N is the number of trees per acre. Hence, the first two moments of the diameter distribution have stand-level interpretations that are common in forestry practice.

Stand average estimates of the first k moments produce a system of k equations with k unknown parameters which can be solved to obtain estimates of the pdf parameters while ensuring compatibility between whole stand and diameter distribution estimates of the stand attributes described by the moment equations.

STAND ATTRIBUTE PREDICTION

Regression equations used to obtain estimates of the first two noncentral moments, and subsequently solve for the parameters of the Weibull distribution, are given in Table 5.

TABLE 5. Equations for prediction of the first and second noncentral moments of the diameter distribution.[a]

$$\ln(B_2) = (A_1/A_2)\ln(B_1) + (b_4/b_3)(1 - A_1/A_2) + (b_5/b_3)(S)(1 - A_1/A_2) \text{(from Table 4)}$$

$$\ln(\overline{d^2} - \bar{d}^2) = b_0 + b_1\ln(b) + b_2\ln(H_d) + b_3(A \bullet N)/1{,}000$$

For before first thinning		For after first thinning	
$b_0 = -13.40824$	$R^2 = 0.8133$	$b_0 = -5.20164$	$R^2 = 0.3726$
$b_1 = 0.45213$	$s^2 = 0.09357$	$b_1 = 0.80773$	$s^2 = 0.2225$
$b_2 = 3.05978$		$b_2 = 0.72383$	
$b_3 = -0.20664$		$b_3 = -0.33560$	

$$\bar{d} = \{B/(0.005454N) - \exp[\ln(\overline{d^2} - \bar{d}^2)]\}^{1/2}$$

$$\ln(Dmin) = 1.19439 + 0.05637[B/(0.005454N)]^{1/2} + 3.04022/(N^{1/2}) - 394.07219/(A \bullet H_d)$$

$R^2 = 0.8251$ $s^2 = 0.02045$

(For all measures except before first thinning where Dmin is set equal to 5.0 inches.)

[a]Where

- A_1 = stand age at beginning of projection period.
- A_2 = stand age at end of projection period.
- A = stand age.
- B_1 = basal area/acre (sq ft) at beginning of projection period.
- B_2 = basal area/acre (sq ft) at end of projection period.
- B = basal area/acre (sq ft)
- S = site index, base age 50 years.
- $\overline{d^2}$ = average squared tree dbh of stand (inches^2).
- $\bar{d}$ = average tree dbh of stand (inches).
- H_d = average height of dominant and codominant trees of stand (feet).
- N = number of trees/acre.
- Dmin = minimum dbh of stand (inches).
- R^2 = coefficient of determination,
- s^2 = mean squared error.
- ln = natural (Naperian) logarithm.

The moment-based system of equations for the three parameter Weibull distribution led to convergence problems and the three parameter Weibull pdf was reduced to the two parameter form using the transformation X = Z – a. That is, the location parameter a was set equal to a constant or predicted outside the system of equations, depending on stand characteristics.

Because independent estimates of average diameter, $\bar{d}$, and average squared diameter, $\overline{d^2}$, often produced illogical crossovers and hence negative variances (i.e., $\overline{d^2} - \bar{d}^2 < 0$), a procedure discussed by Frazier (1981) was used, i.e., the logarithm of the variance of the diameters, $\ln(\overline{d^2} - \bar{d}^2)$, was predicted. Given a value of $\overline{d^2}$ obtained from the estimate of basal area and the estimate of $\ln(\overline{d^2} - \bar{d}^2)$, $\bar{d}$ was determined algebraically.

As only those trees ≥4.5 inches in dbh were tallied, and due to the extremely small variability in minimum stand diameters for the plot data prior to the first thinning, the minimum diameter, Dmin, was set equal to 5.0 inches in stands prior to the first thinning.

Bailey and Dell (1973) state that *a* can be considered the smallest possible diameter in the stand. An approximation to this smallest possible diameter is given by Dmin, the minimum observed diameter on the sample plots. This value is positively biased since Dmin is always greater than or equal to the true smallest diameter in the stand. Thus the value of *a* should most likely be $0 \le a \le$ Dmin. Five values for Dmin were selected and sensitivity analyses conducted. Using values of 0, 1/3(Dmin), 1/2(Dmin), 2/3(Dmin), and Dmin for *a*, and the recovered estimates of *b* and *c*, observed and predicted diameter distributions were compared. As was previously found by Frazier (1981) for thinned loblolly pine stands, preliminary tests with the yellow-poplar data indicated that the *a* parameter of the Weibull distribution could be estimated reasonably well from the minimum stand diameter, Dmin, as

$$a = 0.5(\text{Dmin}).$$

The two equations for the two parameter system are

$$\bar{x} = \int_0^\infty x f(x; b, c)\, dx = b\Gamma(1 + 1/c) \tag{7}$$

$$\overline{x^2} = \int_0^\infty x^2 f(x; b, c)\, dx = b^2\Gamma(1 + 2/c) \tag{8}$$

The estimated variance of the distribution is given by

$$s^2 = \overline{x^2} = \bar{x}^2 = b^2[\Gamma(1 + 2/c) - \Gamma^2(1 + 1/c)] \tag{9}$$

and the coefficient of variation (CV) is estimated by

$$CV = \frac{s}{\bar{x}} = \frac{\left[\Gamma(1 + 2/c) - \Gamma^2(1 + 1/c)\right]^{1/2}}{\Gamma(1 + 1/c)} \tag{10}$$

Given estimates of $\bar{x}$ and $\overline{x^2}$, the coefficient of variation is a function of *c* alone, thus reducing the order of the system. Under this formulation, there exists a unique solution for *c*, and simple iterative techniques for solving one equation in one unknown can be used to obtain a value for *c*. With c known, *b* is solved from $\bar{x} = b\Gamma(1 + 1/c)$, and *a* is estimated with a constant or equation external to the system. In a sense, this is a "hybrid" system in that it combines the parameter-prediction and parameter-recovery systems.

When applying the system, the same stand-level basal area equation is used when deriving diameter distributions and when estimating overall stand basal area in order to ensure compatibility between the two levels of stand detail.

The computer program written by Frazier (1981) to approximate the diameter distributions of unthinned plantations of loblolly pine was used as a framework in the development of the yellow-poplar growth and yield program. Equations to predict stand attributes required by the solution routine, such as mean height of the dominant and codominant trees, number of trees per acre, and individual tree volume, are presented in Table 6.

The total height equation is a slight modification of the one presented by Beck and Della-Bianca (1970) with number of trees per acre replaced by basal area per acre. The tree volume equation is of the same form presented by Beck (1963) and was fitted using weighted least squares procedures.

TABLE 6. Stand attribute prediction equations.[a]

$$\ln(H_d/H) = -0.09675 + (1/D - 1/Dmax)\bullet[3.70051 - 0.02828\ \ln(B) = 138.35633(A^{-1}) + 0.04010(S)]$$

$R^2 = 0.8312$ $s^2 = 0.006037$

$$TVOB = 0.010309 + 0.002399(D^2 \bullet H)$$

$$\ln(B) = b_0 + b_1(A^{-1}) + b_2(S) + b_3(N^{-1})$$

For before first thinning		For after first thinning		For after second thinning	
$b_0 = 4.55808$	$R^2 = 0.6838$	$b_0 = 4.16240$	$R^2 = 0.7404$	$b_0 = 4.24861$	$R^2 = 0.7929$
$b_1 = -31.21173$	$s^2 = 0.02493$	$b_1 = -38.13602$	$s^2 = 0.03980$	$b_1 = -45.83883$	$s^2 = 0.02634$
$b_2 = 0.01324$		$b_2 = 0.01606$		$b_2 = 0.01566$	
$b_3 = -77.35908$		$b_3 = -47.19922$		$b_3 = -37.78880$	

$$\ln(N) = b_0 + b_1(A^{-1}) + b_2(S) + b_3(B^{-1})$$

For before first thinning		For after first thinning		For after second thinning	
$b_0 = 6.433465$	$R^2 = 0.6115$	$b_0 = 6.12444$	$R^2 = 0.7707$	$b_0 = 6.12335$	$R^2 = 0.7213$
$b_1 = 38.24834$	$s^2 = 0.03671$	$b_1 = 59.93859$	$s^2 = 0.06980$	$b_1 = 69.03772$	$s^2 = 0.07113$
$b_2 = -0.01309$		$b_2 = -0.01911$		$b_2 = -0.02083$	
$b_3 = -67.25874$		$b_3 = -73.59987$		$b_3 = -78.12201$	

[a]Where

H_d = average height of dominant and codominant trees of stand (feet).
H = total tree height (feet).
D = dbh (inches).
Dmax = maximum dbh of stand (inches).
B = basal area/acre (sq ft) of stand.
A = age of stand.
S = site index, base age 50 years (feet).
TVOB = total tree cubic-foot volume, outside bark.
N = number of trees/acre of stand.
R^2 = coefficient of determination.
s^2 = mean squared error.
ln = natural (Naperian) logarithm.

THINNING ALGORITHM

Using the equations presented in Table 6, diameter distributions before and after the first thinning were predicted for 10 randomly selected sample plots to observe the "goodness-of-fit" of the system and also to check for logical consistencies which should exist between stand tables for thinned and unthinned conditions.

Although the predicted distributions closely approximated the observed distributions, some discrepancies were present among the stand tables of the thinned and unthinned plots. Predicted numbers of trees increased in some diameter classes after thinning, and, in some instances, the thinned stand table had a larger maximum stand diameter and/or a smaller minimum stand diameter than those in the corresponding unthinned stand table. It was apparent that the diameter distribution predictions before and after a thinning from below could not be carried out independently, but had to be conditioned such that the previously stated inconsistencies could not occur.

As an alternative to two independent predictions, the diameter distribution prior to thinning was predicted, as before, then a proportion of the basal area in each diameter class was removed to simulate the thinning. With this procedure it is impossible for the number of trees in a given class to increase as trees can only be removed from a class. Consequently, minimum diameter can only increase and maximum diameter can only decrease, if they change at all.

A function was defined specifying the amount of basal area to be removed from each diameter class. The following equation form relating the proportion of basal area removed in a diameter class to the ratio of the midpoint diameter of the class to the average squared diameter of the stand was used to "thin" the predicted stand table.

$$P_i = \exp[b_1(d_i^2/\overline{d^2})^{b_2}] \tag{11}$$

where

P_i = proportion of basal area removed from diameter class i,
d_i = midpoint diameter of class i,
$\overline{d^2}$ = average squared diameter of stand, and
b_1, b_2 = coefficients estimated from the data.

As the plot data were taken from stands thinned from below, the removal function "thins" more heavily in the smaller diameter classes than in the larger diameter classes. Equation (11), when fitted, represents the average removal pattern in the data used to estimate the parameters. Separate removal equations were fitted for stands after the first and second thinnings due to the obvious differences in the size-class distributions. Coefficient estimates and fit statistics for the two equations are given in Table 7.

Once the basal area removal functions were defined, the thinning algorithm was as follows:

TABLE 7. Coefficient estimates and fit statistics for the basal area removal function.[a]

$$P_i = \exp[b_1(d_i^2/\overline{d^2})^{b_2}]$$

For first thinning	For second thinning
$b_1 = -0.70407$	$b_1 = -2.61226$
$b_2 = 1.87666$	$b_2 = 2.00627$
$R^2 = 0.5614$	$R^2 = 0.4060$
MSE = 0.0843	MSE = 0.0672

[a]Where

P_i = proportion of basal area removed from diameter class i.
d_i = midpoint diameter of class i.
$\overline{d^2}$ = average squared diameter of class i.
MSE = mean square error.
R^2

$$= 1 - \frac{\sum_{i=1}^{n} (P_i - \hat{P}_i)^2}{\sum_{i=1}^{n} (P_i - \overline{P})^2}$$

$\hat{P}_i$ = predicted value of P_i.
$\overline{P}$ = mean of the P_i values.
n = sample size.

1. Predict the diameter distribution prior to thinning from the Weibull distribution.
2. Starting with the smallest diameter class, remove the proportion of basal area specified by the removal function.
3. Proceed through the diameter classes until the desired level of basal area to be removed is attained.
4. If the required basal area removal is not obtained after the largest diameter class is reached, return to the smallest diameter class and remove the remaining basal area in that class. Proceed in this manner through the diameter classes until the desired level of basal area removal is attained.

This procedure validated fairly well against the observed data where the thinnings from below produced stands that were thinned heavily in the lower diameter classes, and diameter distributions that were frequently left-truncated.

Tree Volume Equations

As yellow-poplar is cut for a variety of products, reliable estimates of volume to any specified merchantable top diameter and/or height limit are essential. Beck (1963) published cubic-foot volume tables for yellow-poplar in the southern Appalachians based on diameter at breast height (dbh) and total tree height. Total height, rather than merchantable height, was used to estimate volume inside and outside bark to 4- and 8-inch top diameter limits. However, merchantability standards change rapidly and it is desirable to have a set of volume estimating equations that are completely general and flexible for obtaining estimates for any specified portion of tree boles. To provide estimates of cubic-foot volume to any desired top diameter or height limit while ensuring that the predicted volumes were logically related, we predicted total stem volume and the ratio of merchantable stem volume to total stem volume for any specified top diameter or height limit according to the methods described by Burkhart (1977) and Cao and Burkhart (1980). Information on the individual tree data analyses, which include taper functions as well as the volume equations, can be found in Knoebel and others (1984).

Computer Program

The original source code for the yellow-poplar growth and yield model was written in FORTRAN Level-G. A new computer program for Windows has been developed and is described below.

INPUT DATA

The input data required by the program are:

- Age at beginning of projection period.
- Age at end of projection period (equal to age at beginning of projection period if no projection desired).
- Site index in feet (base age 50 ft).
- Basal area per acre at beginning of projection period (sq ft).
- Number of trees per acre at beginning of projection period.
- Number of previous thinnings.

Either basal area or number of trees per acre or both must be known. Given one measure of stand density, the other can be predicted from age, site index, and the known measure of stand density from equations fitted to the plot data. For projecting stands, the known number of trees or the number of trees obtained from a previously generated stand table should be entered. When this information is not known, the number of trees must be estimated.

STAND ATTRIBUTE PREDICTION

Given the input data, the following stand attributes are computed.

- Average height of the dominant and codominant trees in feet.
- Minimum diameter in inches.
- Arithmetic mean diameter in inches.
- Quadratic mean diameter in inches.

Stand-level estimates are computed at this point.

- Number of trees per acre.
- Basal area per acre (sq ft).
- Total cubic-foot volume per acre.
- Board-foot volume per acre, International ¼-inch rule for all trees in the 11-inch dbh class and above to an 8-inch top (ob).

To obtain the corresponding stand/stock table, estimates of the Weibull distribution parameters must first be computed.

ESTIMATION OF WEIBULL PARAMETERS

Given the input data and the predicted stand attributes, a computer solution routine developed by Burk and Burkhart (1984) is used to obtain estimates of the Weibull parameters. The routine solves a moment-based three parameter Weibull system of equations where the *a* parameter is predicted independent of the system.

STAND TABLE DERIVATION

Given the parameter estimates, number of trees by diameter class are obtained by multiplying the total number of trees per acre by the proportion of the total number of trees in a given class as determined by the three parameter Weibull cdf. Basal area and cubic-foot volume by diameter class are obtained by numerically integrating the general diameter distribution yield function (6) with $g_i(x)$ equal to $0.005454(dbh_2)$ for basal area and $g_i(x)$ equal to a total cubic-foot volume equation, which is a function of dbh alone, for cubic-foot volume.

The numerical integration is carried out using a solution routine developed by Hafley and others (1982). Board-foot volumes in those diameter classes > 11 inches are obtained according to the procedures described by Beck (1964). First, merchantable cubic-foot volume to an 8-inch top diameter (ob) is computed using the volume equations developed by Knoebel and others (1984). Then, using an equation presented by Beck, a board-foot/cubic-foot ratio, and, subsequently, a board-foot volume is calculated for a tree of a specified dbh. Given the number of trees by diameter class and this calculated board-foot volume per tree, an International 1/4-inch board-foot volume for trees ≥11 inches dbh to an 8-inch top (ob) is computed by diameter class.

The user can substitute any total cubic-foot volume equation desired into the program provided all inputs for the equation are a function of diameter alone. For example, if total height is required in the volume equation, which is the case in this program, then an equation to predict total height as a function of dbh must also be supplied.

In addition to number of trees, basal area, and cubic-foot and board-foot volumes per acre by diameter class, the following stand attributes are also given.

- Input data
- Minimum diameter in inches
- Quadratic mean diameter in inches
- Maximum diameter in inches
- Average height of dominants and codominants in feet
- Total number of trees per acre
- Total basal area per acre in square feet
- Total cubic-foot volume per acre
- Total board-foot volume per acre. International ¼-inch rule for all trees in the 11-inch dbh class and above to an 8-inch top (ob).

THINNING THE STAND TABLE

After the projected stand table and associated summary statistics are printed, the user has the option to thin the stand, in which case a residual basal area must be specified. Basal area is then removed from each diameter class according to the thinning algorithm described previously, until the residual basal area limit is met. The number of trees and the cubic-foot and board-foot volumes removed from a diameter class are obtained from the following equations.

$$Nr_i = Br_i/(0.005454D_i^2)$$
$$CVr_i = (Nr_i/Np_i)CVp_i$$
$$BVr_i = (Nr_i/Np_i)BVp_i$$

where

Nr_i	=	number of trees removed from diameter class i
Np_i	=	number of trees prior to thinning in diameter class i
Br_i	=	basal area removed from diameter class i
D_i	=	midpoint dbh of diameter class i
CVr_i	=	cubic-foot volume removed from diameter class i
CVp_i	=	cubic-foot volume prior to thinning in diameter class i
BVr_i	=	board-foot volume removed from diameter class i
BVp_i	=	board-foot volume prior to thinning in diameter class i.

As with the unthinned stand table, a similar stand attribute summary is given for the thinned stand table.

At this point, the user has the option to "rethin" the original predicted stand table to a different residual basal area. This can be done any number of times, to any level of residual basal area greater than zero and less than or equal to the original stand basal area.

MODEL EVALUATION

Evaluation of Whole Stand Estimates

For each of the 141 sample plots, total basal area and cubic-foot volume per acre were computed by summing across the diameter classes of the generated stand tables. In each case, observed minus predicted basal area and cubic-foot volume per acre were calculated. Summary statistics, as well as an R^2 value, were calculated for the basal area and cubic-foot volume residuals. These values are presented in Tables 8 and 9.

Bias, represented by the mean residual, decreases, and goodness-of-fit, represented by R^2, increases for both basal area and cubic-foot volume for the measurement periods after the first thinning, as opposed to the measurement prior to thinning. This may be due to the fact that the diameter distributions of the stands became smoother and more unimodal after the first thinning. Before the first thinning, diameter distributions were generally irregular and often multimodal, making modeling with a Weibull distribution difficult. As the thinnings "smoothed out" the distributions, the bias and goodness-of-fit generally improved. The smoothing effects of the thinnings are most noticeable with basal area as the parameter recovery solution procedure was conditioned on the basal area, and not on cubic-foot volume.

TABLE 8. Summary statistics for the residual values representing observed minus predicted basal area per acre for the sample plot data.

Measurement period	Number of obser-vations	Minimum residual value[a]	Mean residual value	Mean absolute residual value	Maximum residual value	Standard deviation of residual values	R^{2b}
Before first thinning	1.41e+11	0.07	3.64	3.64	26.45	3.13	0.9902
After first thinning		.02	.67	.67	2.26	.44	.9998
Before second thinning		.03	.73	.73	2.33	.45	.9998
After second thinning		.03	.69	.69	2.19	.47	.9998

[a]Residual value computed as the observed minus the predicted value of the dependent variable.

$r_i = Y_i - \hat{Y}_i$.

[b]The R^2 value was computed as follows:

$$R^2 = 1 - \frac{\sum_{i=1}^{n} r_i^2}{\sum_{i=1}^{n} (Y_i - \overline{Y})^2}$$

where

Y_i = i^{th} observed value of the dependent variable.
$\hat{Y}_i$ = i^{th} predicted value of the dependent variable.
$\overline{Y}$ = mean value of the dependent variable.
r_i = i^{th} residual value, as defined above in footnote a.
n = number of observations.

TABLE 9. Summary statistics for the residual values representing observed minus predicted total cubic-foot volume per acre for the sample plot data.

Measurement period	Number of obser-vations	Minimum residual value[a]	Mean residual value	Mean absolute residual value	Maximum residual value	Standard deviation of residual values	R^2[b]
Before first thinning	1.41e+11	-399.13	206.94	249.21	970.32	232.86	0.9860
After first thinning		-783.53	-80.57	123.09	223.36	164.21	.9898
Before second thinning		-498.23	167.72	194.45	685.67	173.57	.9904
After second thinning		-498.23	151.55	173.94	685.67	151.34	.9920

[a]Residual value computed as the observed minus the predicted value of the dependent variable.
$r_i = Y_i - \hat{Y}_i$.
[b]The R^2 value was computed as follows:

$$R^2 = 1 - \frac{\sum_{i=1}^{n} r_i^2}{\sum_{i=1}^{n} (Y_i - \overline{Y})^2}$$

where

Y_i = i^{th} observed value of the dependent variable.
$\hat{Y}_i$ = i^{th} predicted value of the dependent variable.
$\overline{Y}$ = mean value of the dependent variable.
r_i = i^{th} residual value, as defined above in footnote a.
n = number of observations.

An evaluation of the parameter recovery procedure at the diameter class level was also conducted. Using the plot data and the predicted number of trees obtained from the solution routines, the observed and predicted number of trees by diameter class were computed for each plot.

A Chi-square goodness-of-fit statistic was calculated for each plot before and after the first thinning as well as before and after the second thinning. Calculated Chi-square statistics from the 141 plots exhibited trends similar to those found earlier at the whole stand level in that goodness-of-fit, measured by the Chi-square statistics, improved as the time from the initial measurement and number of thinnings increased. In all cases, the Chi-square goodness-of-fit tests indicated that the predicted diameter distributions were not different from the observed distributions at the 0.2573 significance level (for the poorest fit).

Predicted Stand Tables

To evaluate the prediction system in terms of biological relationships, stand tables were generated for various combinations of ages, site indexes, and basal areas, all well within the ranges of the observed data. The numbers of trees per acre were estimated from stand age, site index, and basal area per acre. In all cases, the stands were assumed to have been previously thinned once. These stand tables are presented in Table 10.

SIZE CLASS DISTRIBUTIONS

For a given site index and stand basal area, as age increases, the number of diameter classes also increases. This increase is always due to the addition of larger, not smaller, diameter classes. There is also a general decrease in the number of trees in the smaller diameter classes and a corresponding increase in the number of trees in the larger diameter classes. Finally, it should be noted that as age increases, total number of trees in the stand decreases, for a given site index.

For a given age and stand basal area, an increasing site index also tends to result in an increasing spread in the diameter distribution. Again, the increase in number of diameter classes is always due to the addition of larger diameter classes. With increasing site index there is also a decreasing number of trees in the smaller diameter classes and an increasing number in the larger classes. As was the case with age, a higher site index leads to a lower total number of trees for the stand at a given age.

For a given age and site index, effects due to varying levels of basal area are also present. An increase in basal area is followed by a slight increase in the number of diameter classes as well as an increase in the total number of trees.

In general, the stand tables demonstrate the expected biological relationships in terms of size class distributions due to factors such as age, site index, and stand density.

VOLUME YIELDS

Total cubic-foot volume yields from the stand tables presented in Table 10 are summarized in Table 11. For a given site index and basal area, as age increases, so does volume, however, the rate of increase decreases

TABLE 10. Predicted stand tables for various combinations of age, site index, and basal area values (for stands thinned once).

SITE INDEX 90

	Basal area (sq ft/acre)											
	70				90				110			
Age (years)	Dbh class (inches)	Number of trees per acre	Basal area (sq ft/acre)	Total cubic-foot volume	Dbh class (inches)	Number of trees per acre	Basal area (sq ft/acre)	Total cubic-foot volume	Dbh class (inches)	Number of trees per acre	Basal area (sq ft/acre)	Total cubic-foot volume
20	3	147.3	7.66	168.21	3	193.3	9.98	219.45	3	207.0	10.73	235.64
	4	178.0	15.48	345.27	4	217.7	18.91	422.20	4	245.2	21.35	476.17
	5	124.5	16.76	378.06	5	151.1	20.34	459.10	5	178.6	24.08	543.03
	6	69.1	13.34	303.40	6	86.1	16.65	378.92	6	106.9	20.70	470.35
	7	32.6	8.56	195.70	7	43.0	11.30	258.51	7	56.0	14.75	337.04
	8	13.5	4.63	106.31	8	19.3	6.62	152.24	8	26.5	9.09	208.62
	9	5.0	2.17	50.05	9	7.9	3.45	79.47	9	11.4	4.98	114.52
	10	1.7	0.90	20.82	10	3.0	1.62	37.43	10	4.6	2.46	56.74
	11	0.5	0.33	7.76	11	1.1	0.70	16.11	11	1.7	1.11	25.70
									12	0.6	0.46	10.74
	Sum	572.1	69.83	1,575.58	Sum	722.4	89.56	2,023.43	Sum	838.6	109.71	2,478.54
30	3	0.6	0.03	0.65	3	1.2	0.07	1.24	3	1.4	0.08	1.48
	4	5.8	0.54	11.91	4	8.9	0.82	17.84	4	10.0	0.93	19.77
	5	17.5	2.46	60.70	5	23.6	3.30	80.18	5	25.6	3.59	85.84
	6	32.7	6.52	174.12	6	40.7	8.09	212.57	6	43.5	8.66	224.44
	7	44.4	11.94	338.09	7	52.7	14.16	394.41	7	56.8	15.25	419.19
	8	45.5	15.85	469.11	8	53.4	18.62	542.31	8	59.3	20.69	594.48
	9	34.8	15.27	468.02	9	42.4	18.61	561.27	9	50.0	21.98	653.75
	10	19.4	10.48	330.33	10	26.0	14.05	435.73	10	33.8	18.28	559.05
	11	7.7	4.97	160.32	11	12.1	7.89	250.33	11	18.1	11.80	369.29
	12	2.0	1.57	51.62	12	4.2	3.23	104.40	12	7.5	5.84	186.17
					13	1.0	0.94	30.89	13	2.4	2.18	70.54
									14	0.6	0.60	19.74
	Sum	210.4	69.64	2,064.84	Sum	266.1	88.78	2,631.17	Sum	308.9	109.86	3,203.74

40	3	0.0	0.0	0.0	3	0.0	0.0	0.0	3	0.0	0.0	0.0
	4	0.0	0.0	0.0	4	0.2	0.02	0.46	4	0.3	0.03	0.57
	5	1.0	0.15	3.74	5	1.8	0.26	6.24	5	2.0	0.29	7.04
	6	3.9	0.78	21.85	6	5.7	1.15	31.40	6	6.2	1.26	33.84
	7	9.2	2.50	75.75	7	12.2	3.32	98.63	7	13.0	3.53	103.32
	8	16.5	5.81	187.36	8	20.4	7.18	227.44	8	21.4	7.53	235.14
	9	23.4	10.38	351.89	9	27.6	12.26	408.44	9	29.1	12.92	424.19
	10	26.3	14.37	507.38	10	30.7	16.73	580.52	10	33.2	18.13	619.59
	11	23.0	15.14	552.40	11	27.5	18.10	649.15	11	31.5	20.76	733.40
	12	15.0	11.70	438.97	12	19.5	15.20	560.20	12	24.5	19.16	695.84
	13	6.9	6.33	243.04	13	10.5	9.61	362.69	13	15.3	13.97	519.41
	14	2.2	2.26	88.63	14	4.2	4.42	170.15	14	7.4	7.85	297.76
					15	1.2	1.42	55.57	15	2.7	3.30	127.45
									16	0.7	1.01	39.46
	Sum	127.4	69.42	2,471.01	Sum	161.4	89.65	3,150.88	Sum	187.4	109.75	3,837.01
50	4	0.0	0.0	0.0	4	0.0	0.0	0.0	4	0.0	0.0	0.0
	5	0.0	0.0	0.0	5	0.1	0.02	0.49	5	0.2	0.02	0.60
	6	0.5	0.10	2.77	6	0.9	0.17	4.79	6	1.0	0.20	5.47
	7	1.8	0.48	14.97	7	2.7	0.73	22.29	7	2.9	0.80	24.31
	8	4.4	1.56	52.02	8	6.0	2.14	69.98	8	6.4	2.26	74.00
	9	8.6	3.82	135.01	9	10.9	4.85	168.39	9	11.3	5.03	174.85
	10	13.6	7.47	276.63	10	16.3	8.97	326.70	10	16.9	9.27	337.96
	11	17.8	11.76	452.73	11	20.7	13.68	518.54	11	21.7	14.37	544.49
	12	18.7	14.66	583.33	12	21.8	17.09	669.20	12	23.8	18.68	731.40
	13	15.3	14.00	572.66	13	18.6	17.10	688.37	13	21.9	20.11	809.68
	14	9.2	9.75	408.51	14	12.5	13.29	547.63	14	16.5	17.56	723.92
	15	3.8	4.67	199.46	15	6.3	7.71	324.23	15	9.9	12.09	508.91
	16	1.0	1.43	62.08	16	2.3	3.19	136.50	16	4.6	6.35	272.37
					17	0.6	0.89	38.79	17	1.6	2.46	106.95
	Sum	94.6	69.71	2,760.18	Sum	119.7	89.82	3,515.90	Sum	138.5	109.19	4,314.91

TABLE 10. *Continued.*

SITE INDEX 110

	Basal area (sq ft/acre)											
	70				90				110			
Age (years)	Dbh class (inches)	Number of trees per acre	Basal area (sq ft/acre)	Total cubic-foot volume	Dbh class (inches)	Number of trees per acre	Basal area (sq ft/acre)	Total cubic-foot volume	Dbh class (inches)	Number of trees per acre	Basal area (sq ft/acre)	Total cubic-foot volume
20	3	32.7	1.79	40.20	3	47.5	2.58	57.44	3	52.8	2.86	63.38
	4	82.1	7.32	176.30	4	104.7	9.31	222.38	4	114.9	10.21	242.46
	5	96.9	13.25	334.82	5	116.4	15.88	398.47	5	129.1	17.64	439.90
	6	81.2	15.84	414.17	6	96.2	18.77	487.02	6	110.1	21.51	554.77
	7	52.8	13.94	373.63	7	64.4	17.03	452.97	7	77.4	20.50	541.74
	8	27.5	9.47	258.64	8	36.1	12.46	337.45	8	46.4	16.01	430.92
	9	11.7	5.10	141.28	9	17.3	7.54	207.34	9	24.1	10.53	287.43
	10	4.1	2.21	61.95	10	7.2	3.85	107.05	10	11.0	5.93	163.79
	11	1.2	0.78	22.06	11	2.6	1.68	47.05	11	4.4	2.90	80.78
					12	0.8	0.63	17.77	12	1.6	1.24	34.80
									13	0.5	0.47	13.19
	Sum	390.3	69.71	1,823.05	Sum	493.2	89.72	2,334.96	Sum	572.4	109.78	2,853.16
30	3	0.0	0.0	0.0	3	0.0	0.0	0.0	3	0.0	0.0	0.0
	4	0.5	0.05	1.07	4	1.0	0.09	2.07	4	1.2	0.11	2.51
	5	2.8	0.40	10.75	5	4.5	0.64	16.71	5	5.2	0.73	18.85
	6	7.9	1.60	47.10	6	11.1	2.23	64.52	6	12.2	2.44	69.72
	7	15.5	4.21	133.63	7	19.8	5.36	167.75	7	21.2	5.73	176.79
	8	23.5	8.27	278.39	8	28.3	9.93	329.18	8	30.0	10.53	344.21
	9	28.4	12.59	443.72	9	33.1	14.66	508.53	9	35.5	15.73	538.27
	10	27.3	14.86	543.21	10	31.9	17.38	625.38	10	35.5	19.36	687.02
	11	20.3	13.35	502.62	11	25.0	16.43	609.22	11	29.8	19.60	716.60
	12	11.4	8.85	341.66	12	15.6	12.19	463.43	12	20.7	16.17	606.47
	13	4.6	4.17	164.45	13	7.6	6.95	269.73	13	11.7	10.74	411.32
	14	1.3	1.34	53.68	14	2.8	2.96	117.21	14	5.3	5.64	220.01
					15	0.8	0.92	36.99	15	1.9	2.30	91.12
									16	0.5	0.71	28.63
	Sum	143.6	69.68	2,320.28	Sum	181.5	89.76	3,210.73	Sum	210.7	109.80	3,911.52

40	4	0.0	0.0	0.0	4	0.0	0.0	0.0	4	0.0	0.0	0.0
	5	0.0	0.0	0.0	5	0.0	0.0	0.0	5	0.1	0.02	0.44
	6	0.3	0.06	1.93	6	0.6	0.12	3.64	6	0.7	0.14	4.26
	7	1.2	0.33	10.97	7	1.9	0.53	17.45	7	2.2	0.59	19.30
	8	3.1	1.10	39.28	8	4.4	1.56	55.82	8	4.8	1.69	59.39
	9	6.2	2.76	105.04	9	8.1	3.61	137.12	9	8.5	3.80	142.15
	10	10.2	5.62	224.18	10	12.5	6.88	274.48	10	13.0	7.14	280.84
	11	14.2	9.40	390.26	11	16.6	11.02	457.70	11	17.3	11.47	469.50
	12	16.4	12.86	552.22	12	18.9	14.83	637.01	12	20.1	15.79	668.66
	13	15.3	14.08	622.21	13	18.0	16.56	731.94	13	20.1	18.54	807.59
	14	11.2	11.92	539.88	14	14.1	15.03	680.55	14	17.2	18.31	817.15
	15	6.1	7.46	345.13	15	8.8	10.77	498.27	15	12.2	14.92	680.40
	16	2.4	3.27	154.08	16	4.3	5.90	277.91	16	7.1	9.81	455.84
	17	0.6	0.94	45.13	17	1.5	2.37	113.64	17	3.2	5.07	239.35
									18	1.1	2.0	95.63
	Sum	87.2	69.81	3,030.30	Sum	109.7	89.17	3,885.54	Sum	127.6	109.28	4,740.51
50	5	0.0	0.0	0.0	5	0.0	0.0	0.0	5	0.0	0.0	0.0
	6	0.0	0.0	0.0	6	0.0	0.0	0.0	6	0.0	0.0	0.0
	7	0.0	0.0	0.0	7	0.2	0.05	1.84	7	0.2	0.07	2.17
	8	0.4	0.14	5.37	8	0.7	0.25	9.05	8	0.8	0.28	10.10
	9	1.1	0.51	20.43	9	1.7	0.77	30.40	9	1.9	0.83	32.50
	10	2.5	1.39	58.80	10	3.5	1.91	79.67	10	3.6	2.00	82.60
	11	4.7	3.10	137.65	11	6.0	3.96	173.30	11	6.1	4.08	176.03
	12	7.4	5.85	270.07	12	8.9	7.07	321.42	12	9.1	7.21	323.65
	13	10.1	9.36	446.44	13	11.8	10.89	511.89	13	12.1	11.19	518.77
	14	11.7	12.51	614.25	14	13.4	14.39	695.72	14	14.2	15.18	724.17
	15	11.1	13.63	685.48	15	13.1	16.03	794.10	15	14.5	17.85	872.28
	16	8.4	11.63	597.43	16	10.5	14.68	742.99	16	12.8	17.87	892.44
	17	4.7	7.40	387.26	17	6.8	10.71	552.23	17	9.5	14.92	759.26
	18	1.9	3.31	175.93	18	3.4	5.99	313.94	18	5.8	10.11	523.37
					19	1.3	2.45	130.61	19	2.8	5.40	283.54
									20	1.0	2.19	116.62
	Sum	64.0	68.82	3,399.11	Sum	81.3	89.14	4,357.15	Sum	94.5	109.19	5,317.48

TABLE 10. Continued.

SITE INDEX 130

Age (years)	70: Dbh class (inches)	70: Number of trees per acre	70: Basal area (sq ft/acre)	70: Total cubic-foot volume	90: Dbh class (inches)	90: Number of trees per acre	90: Basal area (sq ft/acre)	90: Total cubic-foot volume	110: Dbh class (inches)	110: Number of trees per acre	110: Basal area (sq ft/acre)	110: Total cubic-foot volume
20	3	4.7	0.27	5.88	3	7.9	0.44	9.69	3	9.4	0.53	11.36
	4	22.3	2.03	50.57	4	31.5	2.86	70.22	4	35.3	3.20	77.81
	5	43.1	5.98	161.82	5	54.7	7.58	202.70	5	59.8	8.27	219.14
	6	56.3	11.11	318.96	6	67.2	13.25	375.91	6	73.2	14.44	405.71
	7	55.4	14.78	442.78	7	64.6	17.24	510.56	7	71.8	19.18	562.49
	8	42.3	14.67	454.23	8	50.4	17.49	535.29	8	58.6	20.35	616.58
	9	25.3	11.03	350.37	9	32.3	14.14	443.89	9	40.3	17.66	548.77
	10	11.7	6.29	203.96	10	17.1	9.19	294.54	10	23.5	12.68	402.19
	11	4.2	2.71	89.31	11	7.4	4.82	157.18	11	11.6	7.58	244.57
	12	1.1	0.87	29.22	12	2.6	2.04	67.55	12	4.9	3.79	123.88
					13	0.8	0.70	23.36	13	1.7	1.58	52.35
									14	0.5	0.55	18.46
	Sum	266.3	69.75	2,107.10	Sum	336.5	89.76	2,690.90	Sum	390.5	109.79	3,283.32
30	4	0.0	0.0	0.0	4	0.0	0.0	0.0	4	0.0	0.0	0.0
	5	0.2	0.03	0.69	5	0.4	0.05	1.46	5	0.5	0.07	1.81
	6	0.9	0.19	5.90	6	1.6	0.32	9.93	6	1.9	0.38	11.44
	7	2.7	0.75	25.67	7	4.1	1.12	37.66	7	4.6	1.24	41.29
	8	5.9	2.09	77.04	8	8.0	2.82	102.37	8	8.6	3.03	108.50
	9	11.3	4.58	178.19	9	12.9	5.73	219.72	9	13.5	6.03	228.14
	10	14.8	8.14	331.39	10	17.6	9.66	387.47	10	18.4	10.11	400.06
	11	17.9	11.83	500.03	11	20.7	13.66	568.49	11	21.9	14.48	594.62
	12	17.7	13.89	605.58	12	20.5	16.11	691.93	12	22.6	17.72	750.90
	13	14.0	12.84	574.52	13	17.0	15.65	689.77	13	20.0	18.41	800.72
	14	8.5	9.01	412.50	14	11.5	12.25	552.16	14	15.0	16.01	712.41
	15	3.3	4.60	214.74	15	6.2	7.53	346.11	15	9.4	11.47	520.33
	16	1.2	1.62	77.10	16	2.6	3.53	164.95	16	4.8	6.63	305.94
					17	0.8	1.22	57.82	17	1.9	3.02	141.63
									18	0.6	1.06	50.37
	Sum	97.9	69.57	3,003.36	Sum	123.8	89.64	3,829.82	Sum	143.7	109.66	4,668.17

40	5	0.0	0.0	0.0	5	0.0	0.0	0.0	5	0.0	0.0	0.0
	6	0.0	0.0	0.0	6	0.0	0.0	0.0	6	0.0	0.0	0.0
	7	0.0	0.0	0.0	7	0.1	0.03	1.08	7	0.1	0.04	1.33
	8	0.2	0.08	3.20	8	0.4	0.15	5.81	8	0.5	0.18	6.69
	9	0.7	0.31	13.10	9	1.1	0.50	20.62	9	1.2	0.56	22.61
	10	1.6	0.90	39.75	10	2.3	1.29	56.36	10	2.5	1.38	59.57
	11	3.2	2.11	97.65	11	4.2	2.79	127.58	11	4.4	2.91	131.29
	12	5.3	4.19	202.13	12	6.6	5.21	247.49	12	6.8	5.33	250.51
	13	7.7	7.16	357.24	13	9.2	8.48	416.79	13	9.4	8.65	420.25
	14	9.8	10.46	536.96	14	11.2	12.04	609.23	14	11.6	12.45	622.08
	15	10.4	12.83	674.74	15	12.0	14.75	765.30	15	12.9	15.78	808.83
	16	9.2	12.82	689.07	16	11.0	15.31	811.74	16	12.5	17.45	913.90
	17	6.4	10.04	550.36	17	8.4	13.12	709.56	17	10.5	16.54	883.27
	18	3.3	5.88	327.67	18	5.1	9.02	495.94	18	7.5	13.17	715.60
	19	1.2	2.43	137.33	19	2.5	4.79	267.42	19	4.4	8.60	474.39
					20	0.9	1.89	106.76	20	2.1	4.47	250.26
									21	0.8	1.80	101.82
	Sum	59.2	69.21	3,629.23	Sum	75.0	89.36	4,641.67	Sum	87.0	109.32	5,662.39
50	6	0.0	0.0	0.0	6	0.0	0.0	0.0	6	0.0	0.0	0.0
	7	0.0	0.0	0.0	7	0.0	0.0	0.0	7	0.0	0.0	0.0
	8	0.0	0.0	0.0	8	0.0	0.0	0.0	8	0.0	0.0	0.0
	9	0.0	0.0	0.0	9	0.0	0.0	0.0	9	0.1	0.05	2.17
	10	0.2	0.09	4.21	10	0.3	0.17	7.53	10	0.3	0.19	8.41
	11	0.4	0.30	14.65	11	0.7	0.48	23.02	11	0.8	0.52	24.64
	12	1.0	0.80	40.57	12	1.4	1.15	57.71	12	1.5	1.20	59.80
	13	1.9	1.79	94.67	13	2.6	2.38	124.43	13	2.6	2.44	125.81
	14	3.3	3.50	191.50	14	4.1	4.38	236.10	14	4.1	4.41	235.05
	15	4.9	6.03	338.91	15	5.8	7.17	397.74	15	5.8	7.19	394.11
	16	6.5	9.07	522.45	16	7.5	10.44	593.90	16	7.6	10.57	594.20
	17	7.4	11.73	690.75	17	8.5	13.39	778.47	17	8.8	13.95	801.66
	18	7.2	12.71	763.34	18	8.4	14.86	840.91	18	9.3	16.35	957.88
	19	5.7	11.14	680.43	19	7.1	13.94	840.87	19	8.5	16.75	998.39
	20	3.5	7.53	467.55	20	4.9	10.73	657.53	20	6.8	14.70	890.09
	21	1.6	3.72	234.36	21	2.7	6.54	406.60	21	4.5	10.79	662.73
					22	1.2	3.03	190.80	22	2.5	6.44	400.73
									23	1.1	3.03	190.57
	Sum	43.5	68.41	4,043.39	Sum	55.2	88.66	5,195.62	Sum	64.2	108.57	6,346.23

TABLE 11. Total cubic-foot volume yields for various combinations of site index, age, and basal area values of yellow-poplar stands thinned one time.

Site index and age (years)	Basal area (sq ft/acre)		
	70	90	110
Site index 90	-- cubic feet ---		
20304050	1576206524712760	2023263131513516	2479320438374315
Site index 110			
20304050	1823252030303399	2335321138864357	2853391247415317
Site index 130			
20304050	2107300336294043	2691383046425196	3283466856626346

with age. When age and site index are fixed, an increase in basal area results in an increase in total cubic-foot volume which is fairly constant across the basal area classes. Higher volumes are also associated with higher site indexes. It should be noted that stands of higher site indexes have correspondingly larger volume differences between age periods than those of lower sites. The trends in total cubic-foot volume reflected in Table 11 are generally in agreement with known biological relationships.

Effect of Thinning Regime on Yield

Six thinning regimes were outlined to determine the effects of thinning on volume yields and to answer the following questions:

1. How does the weight of thinning affect yield?
2. How does the number of thinnings affect yield?
3. How does the timing of thinnings affect yield?

WEIGHT OF THINNING

To describe the influence of the weight of thinning on volume yields, two thinning regimes were specified, differing only in the amount of basal area removed at each thinning. Both regimes were modeled at three levels of site index to describe how the trends due to the thinning regimes are affected on "poor," "average," and "good" sites. The regimes are as follows:

Initial conditions: Site index (base age 50) = 80, 110, 140 ft
Initial age = 20 years
Initial basal area = 80 sq ft/acre.

Regime 1: Thin to 50 sq ft/acre at age 20
Project to age 40 and thin to 70 sq ft/acre
Project to age 50 and thin to 80 sq ft/acre
Project to age 80.

Regime 2: Thin to 65 sq ft/acre at age 20
Project to age 40 and thin to 90 sq ft/acre
Project to age 50 and thin to 110 sq ft/acre
Project to age 80.

Stand-level summaries of total cubic-foot volume (ob) and board-foot volume yields per acre are given in Tables 12 and 13. Board-foot volume per acre is International 1/4-inch rule for all trees in the 11-inch dbh class and above to an 8-inch top diameter (ob). In general, total cubic-foot and board-foot volume yields decrease as thinning weight increases. Due to the definition and structure of the thinning algorithm, for all three site indexes, the diameter distributions for the heavily thinned stands are shifted toward the larger diameter classes, as evidenced by the minimum, quadratic mean, and maximum diameters given for the final stand tables at age 80. The stand tables from regime 1 had less trees, basal area, total cubic-foot volume, and board-foot volume per acre. The differences in volume yields due to weight of thinning tend to increase with increasing site index.

NUMBER OF THINNINGS

To demonstrate the effects of number of thinnings on volume yields, two additional thinning schedules were outlined. These regimes differ from regimes 1 and 2 only in that the stands are thinned once. Given the same initial conditions as before, including the three levels of site index, regimes 3 and 4 are as follows:

Regime 3: Project to age 40 and thin to 70 sq ft/acre
Project to age 80.

Regime 4: Project to age 40 and thin to 90 sq ft/acre
Project to age 80.

Stand-level summaries of total cubic-foot volume (ob) and board-foot volume yields per acre are given in Tables 14 and 15. Board-foot volume per acre is International 1/4-inch rule for all trees in the 11-inch dbh class and above to an 8-inch top diameter (ob). Upon comparison of yields from regimes 1 and 3, the additional thinnings in regime 1 resulted in increased cubic-foot and board-foot yields throughout the rotation at the low site. At the high site, regime 3 had the larger cubic-foot and board-foot volume yields. There were small differences in volume yields for the moderate sites. Similar trends are apparent when comparing yields from regimes 2 and 4. Because the coefficients for the basal area and cubic-foot volume projection equations in the "two-or-more" thinning case produce greater basal area and volume growth, these trends are as expected.

TABLE 12. Stand-level summaries of total cubic-foot and board-foot volume yields per acre for thinning regime 1.

Site index and age (yrs.)	Before thinning: Number of trees per acre	Basal area (sq ft/ac)	Total vol. (ob) (cu ft/ac)	Total volume (bd ft/ac)	After thinning: Number of trees per acre	Basal area (sq ft/ac)	Total vol. (ob) (cu ft/ac)	Total volume (bd ft/ac)	Volume removed (cu ft/ac)	Volume removed (bd ft/ac)	Total vol. production (cu ft/ac)	Total vol. production (bd ft/ac)[a]
Site index 80												
20	638	80	1,671	0	334	50	1,037	0	634	0	1,671	0
30	334	79	2,058	141							2,692	141
40	334	99	2,913	822	175	70	2,154	822	759	0	3,547	822
50	175	98	3,375	6,123	122	80	2,825	6,072	550	51	4,768	6,123
60	122	104	3,941	12,517							5,884	12,568
70	122	125	4,988	18,129							6,931	18,180
80	122	143	5,981	23,466							7,924	23,517
Site index 110												
20	431	80	2,098	56	203	50	1,335	56	763	0	2,098	56
30	203	90	3,226	3,802							3,989	3,802
40	203	121	4,996	11,495	82	70	3,065	10,803	1,931	692	5,759	11,495
50	82	98	4,799	19,648	59	80	3,986	17,109	813	2,541	7,493	20,340
60	59	104	5,549	26,029							9,056	29,260
70	59	125	7,076	34,994							10,583	38,225
80	59	144	8,431	43,242							11,938	46,473
Site index 140												
20	291	80	2,519	1,546	122	50	1,646	1,530	813	16	2,519	1,546
30	122	103	4,761	15,958							5,634	15,974
40	122	148	8,066	33,982	40	70	4,034	18,999	4,032	14,983	8,939	33,998
50	40	98	6,259	32,461	29	80	5,168	27,347	1,091	5,114	11,164	47,460
60	29	104	7,211	40,346							13,207	60,459
70	29	125	9,169	53,113							15,165	73,226
80	29	144	10,965	65,038							16,961	85,151

[a] Board-foot volume per acre; International ¼-inch rule, for all trees in the 11-inch dbh class and above to an 8-inch top diameter (ob).

TABLE 13. *Stand-level summaries of total cubic-foot and board-foot volume yields per acre for thinning regime 2.*

	Before thinning				After thinning							
Site index and age (yrs.)	Number of trees per acre	Basal area (sq ft/ac)	Total vol. (ob) (cu ft/ac)	Total volume (bd ft/ac)	Number of trees per acre	Basal area (sq ft/ac)	Total vol. (ob) (cu ft/ac)	Total volume (bd ft/ac)	Volume removed (cu ft/ac)	Volume removed (bd ft/ac)	Total vol. production (cu ft/ac)	Total vol. production (bd ft/ac)[a]
Site index 80												
20	683	80	1,671	0	450	65	1,348	0	323	0	1,671	0
30	450	94	2,370	162							2,693	162
40	450	113	3,276	541	280	90	2,714	541	562	0	3,599	541
50	280	120	3,998	4,121	234	110	3,725	4,121	273	0	4,883	4,121
60	234	135	4,872	9,822							6,030	9,822
70	234	156	5,947	15,111							7,105	15,111
80	234	174	6,988	20,285							8,146	20,285
Site index 110												
20	431	80	2,098	56	280	65	1,725	56	373	0	2,098	56
30	280	108	3,753	3,434							4,126	3,434
40	280	139	5,620	10,061	126	90	3,876	10,061	1,744	0	5,993	10,061
50	126	120	5,723	20,958	107	110	5,307	20,305	416	653	7,840	20,958
60	107	135	7,028	29,744							9,561	30,397
70	107	156	8,524	38,245							11,057	38,898
80	107	174	9,903	46,036							12,436	46,689
Site index 140												
20	291	80	2,519	1,546	174	65	2,110	1,546	409	0	2,519	1,546
30	174	123	5,504	15,840							5,913	15,840
40	174	169	8,966	34,130	63	90	5,086	22,695	3,880	11,435	9,375	34,130
50	63	120	7,505	36,706	55	110	6,926	34,242	579	2,464	11,794	48,141
60	55	135	9,222	47,941							14,090	61,840
70	55	157	11,244	60,348							16,112	74,247
80	55	175	13,067	71,842							17,935	85,741

[a] Board-foot volume per acre; International ¼-inch rule, for all trees in the 11-inch dbh class and above to an 8-inch top diameter (ob).

TABLE 14. Stand-level summaries of total cubic-foot and board-foot volume yields per acre for thinning regime 3.

	Before thinning				After thinning							
Site index and age (yrs.)	Number of trees per acre	Basal area (sq ft/ac)	Total vol. (ob) (cu ft/ac)	Total volume (bd ft/ac)	Number of trees per acre	Basal area (sq ft/ac)	Total vol. (ob) (cu ft/ac)	Total volume (bd ft/ac)	Volume removed (cu ft/ac)	Volume removed (bd ft/ac)	Total vol. production (cu ft/ac)	Total vol. production (bd ft/ac)[a]
Site index 80												
20	638	80	1,671	0							1,671	0
30	420	108	2,804	285							2,804	285
40	333	126	3,747	3,393	128	70	2,203	3,136	1,544	257	3,747	3,393
50	128	86	3,015	7,304							4,559	7,561
60	128	99	3,701	10,790							5,245	11,047
70	128	109	4,354	14,112							5,898	14,369
80	128	117	4,821	16,599							6,365	16,856
Site index 110												
20	431	80	2,098	56							2,098	56
30	306	124	4,224	6,610							4,224	6,610
40	248	154	6,176	17,969	51	70	3,108	13,094	3,068	4,875	6,176	17,969
50	51	93	4,710	22,086							7,778	26,961
60	51	113	6,066	30,268							9,134	35,143
70	51	130	7,393	38,081							10,461	42,956
80	51	144	8,501	45,029							11,569	49,904
Site index 140												
20	291	80	2,519	1,546							2,519	1,546
30	221	141	6,029	19,030							6,029	19,030
40	181	187	9,488	40,937	22	70	3,922	21,449	5,566	19,448	9,488	40,937
50	22	101	6,630	38,854							12,196	58,342
60	22	129	9,121	55,594							14,687	75,082
70	22	154	11,449	71,733							17,015	91,221
80	22	176	13,587	86,690							19,153	106,178

[a] Board-foot volume per acre; International ¼-inch rule, for all trees in the 11-inch dbh class and above to an 8-inch top diameter (ob).

TABLE 15. *Stand-level summaries of total cubic-foot and board-foot volume yields per acre for thinning regime 4.*

Site index and age (yrs.)	Before thinning: Number of trees per acre	Before thinning: Basal area (sq ft/ac)	Before thinning: Total vol. (ob) (cu ft/ac)	Before thinning: Total volume (bd ft/ac)	After thinning: Number of trees per acre	After thinning: Basal area (sq ft/ac)	After thinning: Total vol. (ob) (cu ft/ac)	After thinning: Total volume (bd ft/ac)	Volume removed (cu ft/ac)	Volume removed (bd ft/ac)	Total vol. production (cu ft/ac)	Total vol. production (bd ft/ac)[a]
Site index 80												
20	638	80	1,671	0							1,671	0
30	420	108	2,804	285							2,804	285
40	333	126	3,747	3,393	181	90	2,788	3,393	959	0	3,747	3,393
50	181	105	3,659	7,180							4,618	7,180
60	181	117	4,302	10,217							5,261	10,217
70	181	126	4,838	12,875							5,797	12,875
80	181	133	5,335	15,287							6,294	15,287
Site index 110												
20	431	80	2,098	56							2,098	56
30	306	124	4,224	6,610							4,224	6,610
40	248	154	6,176	17,969	78	90	3,909	15,572	2,267	2,397	6,176	17,969
50	78	114	5,635	24,774							7,902	27,171
60	78	134	7,059	32,879							9,326	35,276
70	78	150	8,369	40,490							10,636	42,887
80	78	163	9,466	46,884							11,733	49,281
Site index 140												
20	291	80	2,519	1,546							2,519	1,546
30	221	141	6,029	19,030							6,029	19,030
40	181	187	9,488	40,937	32	90	5,010	26,620	4,478	14,317	9,488	40,937
50	32	124	8,059	45,520							12,537	59,837
60	32	153	10,698	63,142							15,176	77,459
70	32	178	13,133	79,378							17,611	93,695
80	32	199	15,291	94,291							19,769	108,608

[a] Board-foot volume per acre: International ¼-inch rule, for all trees in the 11-inch dbh class and above to an 8-inch top diameter (ob).

The faster growth rate associated with stands thinned two or more times has a greater effect at the low site index. For the low site index, the final stand tables showed the stand thinned more than once (regime 1) to have a diameter distribution with larger trees than the stand thinned only once. While it has fewer trees, the stand thinned three times has a higher basal area, cubic-foot volume, and board-foot volume. At the average site index, the stand tables from the two regimes are very similar in all respects. Finally at the high site index, the stand thinned only once has larger diameter trees, as well as greater numbers of trees, basal area, and cubic-foot and board-foot volumes. Similar trends were observed upon comparison of the stand tables from regimes 2 and 4.

TIMING OF THINNING

To illustrate the effect of timing of thinnings on volume yields, two thinning regimes were specified differing only in the time at which the thinnings occurred.

Given the same initial conditions and the three levels of site index, regimes 5 and 6 are given as:

Regime 5: Thin to 70 sq ft/acre at age 20
Project to age 30 and thin to 80 sq ft/acre
Project to age 40 and thin to 90 sq ft/acre
Project to age 80.

Regime 6: Thin to 70 sq ft/acre at age 20
Project to age 40 and thin to 80 sq ft/acre
Project to age 50 and thin to 90 sq ft/acre
Project to age 80.

Stand-level summaries of total cubic-foot volume (ob) and board-foot volume yields per acre, where again, board-foot volume per acre is International ¼-inch rule for all trees in the 11-inch dbh class and above to an 8-inch top diameter (ob), are given in Tables 16 and 17. The earlier thinnings of regime 5 resulted in greater cubic-foot and board-foot yields for the low and moderate site indexes. For the high site index, total cubic-foot and board-foot productions are similar for both the early and late thinnings. The differences in yields due to timing of thinnings tend to decrease as site index increases. For the low site index in particular, early thinnings result in substantial increases in both board-foot and cubic-foot yields.

Based on the final stand tables, the earlier thinnings of regime 5 resulted in greater numbers of trees, basal area, and cubic-foot and board-foot volumes per acre for all site indexes. In addition, the diameter distributions for the stands from regime 5 are shifted slightly toward larger diameter classes than those associated with the stands of regime 6 which were thinned at a later time. This trend becomes more pronounced as site index increases.

In general, as the weight of thinning increased, cubic-foot and board-foot volume yields decreased. The differences due to weight tended to be greater as site index increased. Additional thinnings resulted in greater volume yields, and as site index increased, the trends due to the number of thinnings reversed. Finally, early thinnings produced higher volume yields than the late thinnings—the differences in yields being smaller for the higher site index values. In the six thinning regimes, the differences in total cubic-foot and board-foot yields, as well as the corresponding basal areas and numbers of trees per acre, throughout the rotations were different due to changes in stand structures attributable to the weight, number, and timing of the thinnings.

TABLE 16. *Stand-level summaries of total cubic-foot and board-foot volume yields per acre for thinning regime 5.*

Site index and age (yrs.)	Before thinning: Number of trees per acre	Before thinning: Basal area (sq ft/ac)	Before thinning: Total vol. (ob) (cu ft/ac)	Before thinning: Total volume (bd ft/ac)	After thinning: Number of trees per acre	After thinning: Basal area (sq ft/ac)	After thinning: Total vol. (ob) (cu ft/ac)	After thinning: Total volume (bd ft/ac)	Volume removed (cu ft/ac)	Volume removed (bd ft/ac)	Total vol. production (cu ft/ac)	Total vol. production (bd ft/ac)[a]
Site index 80												
20	638	80	1,671	0	507	70	1,457	0	214	0	1,671	0
30	507	99	2,464	150	322	80	2,054	150	410	0	2,678	150
40	322	118	3,529	2,220	192	90	2,796	2,220	733	0	4,153	2,220
50	192	120	4,143	9,136							5,500	9,136
60	192	145	5,425	15,552							6,782	15,552
70	192	166	6,541	21,358							7,898	21,358
80	192	184	7,507	26,511							8,864	26,511
Site index 110												
20	431	80	2,098	56	317	70	1,852	56	246	0	2,098	56
30	317	113	3,893	3,091	160	80	2,897	3,091	996	0	4,139	3,091
40	160	118	4,985	14,602	99	90	3,927	14,168	1,058	434	6,227	14,602
50	99	120	5,813	23,910							8,113	24,344
60	99	145	7,573	33,745							9,873	34,179
70	99	166	9,178	42,801							11,478	43,235
80	99	184	10,578	50,936							12,878	51,370
Site index 140												
20	291	80	2,519	1,546	200	70	2,256	1,546	263	0	2,519	1,546
30	200	129	5,692	14,977	87	80	3,729	13,795	1,963	1,182	5,955	14,977
40	87	118	6,552	28,825	56	90	5,120	23,689	1,432	5,136	8,778	30,007
50	56	120	7,511	38,042							11,229	44,360
60	56	145	9,911	52,084							13,569	58,402
70	56	166	11,936	64,595							15,594	70,913
80	56	184	13,784	76,065							17,442	82,383

[a] Board-foot volume per acre; International ¼-inch rule, for all trees in the 11-inch dbh class and above to an 8-inch top diameter (ob).

TABLE 17. Stand-level summaries of total cubic-foot and board-foot volume yields per acre for thinning regime 6.

Site index and age (yrs.)	Before thinning: Number of trees per acre	Before thinning: Basal area (sq ft/ac)	Before thinning: Total vol. (ob) (cu ft/ac)	Before thinning: Total volume (bd ft/ac)	After thinning: Number of trees per acre	After thinning: Basal area (sq ft/ac)	After thinning: Total vol. (ob) (cu ft/ac)	After thinning: Total volume (bd ft/ac)	Volume removed (cu ft/ac)	Volume removed (bd ft/ac)	Total vol. production (cu ft/ac)	Total vol. production (bd ft/ac)[a]
Site index 80												
20	638	80	1,671	0	507	70	1,457	0	214	0	1,671	0
30	507	99	2,464	150							2,678	150
40	507	118	3,347	416	242	80	2,422	416	925	0	3,561	416
50	242	109	3,674	4,230	167	90	3,118	4,230	556	0	4,813	4,230
60	167	114	4,256	10,900							5,951	10,900
70	167	135	5,254	16,240							6,949	16,240
80	167	153	6,214	21,362							7,909	21,362
Site index 110												
20	431	80	2,098	56	317	70	1,852	56	246	0	2,098	56
30	317	113	3,893	3,091							4,139	3,091
40	317	144	5,705	8,854	113	80	3,440	8,854	2,265	0	5,951	8,854
50	113	109	5,219	19,273	81	90	4,398	17,673	821	1,600	7,730	19,273
60	81	114	5,980	26,301							9,312	27,901
70	81	135	7,535	35,095							10,867	36,695
80	81	154	8,849	42,870							12,181	44,470
Site index 140												
20	291	80	2,519	1,546	200	70	2,256	1,546	263	0	2,519	1,546
30	200	129	5,692	14,977							5,955	14,977
40	200	175	9,234	33,154	57	80	4,559	20,187	4,675	12,967	9,497	33,154
50	57	109	6,908	33,755	42	90	5,780	28,888	1,128	4,867	11,846	46,722
60	42	114	7,845	41,762							13,911	59,596
70	42	135	9,819	53,946							15,885	71,780
80	42	154	11,584	65,155							17,650	82,989

[a] Board-foot volume per acre; International ¼-inch rule, for all trees in the 11-inch dbh class and above to an 8-inch top diameter (ob).

In all of these comparisons, only the volume in specified size classes was considered; i.e., no consideration was given to the impact of thinning on the quality of the residual stand. When performing in-depth economic analyses of thinning alternatives, quality, as well as volume, relationships should be considered.

DISCUSSION

Model Limitations and Recommendations

Although the growth and yield model produced logical and consistent results, there are certain limitations in the prediction system. First, due to the structure of the data set, it was not possible to fit an equation to project basal area prior to the first thinning. At measurement periods 1 and 2, all stands were thinned. Thus no data were available on basal area growth in unthinned stands. Until such data become available, the stand level equation for basal area prediction after the first thinning can be used as the best approximation in such cases. Similarly, data were available for stands thinned up to two times. For stands thinned more than twice, the equation for stands based on two thinnings was substituted.

Finally, there were no data on tree mortality. This represents a problem primarily for the unthinned stand table projections. Because of the thinnings made every five years, mortality was virtually nonexistent in the thinned stands. This may not be expected operationally, as repeated thinnings, as well as the thinning operations, can cause damage and death to the residual trees. However, based on the data used in this study, one can only assume no mortality when projecting the stands through time following thinnings. For unthinned stand projection, number of trees must be predicted from the projected age, site index, and basal area.

One recommended area for improvement in this study concerns the development of an appropriate stand-level growth and yield model. Using two sets of coefficients for the Sullivan and Clutter simultaneous growth and yield model-one for stands after one thinning and a second for stands after two thinnings, might suggest that the model form is an over-simplification of reality. The development of a generalized growth/growing stock theory that considers the changes in the relationships brought about by thinning in the population would represent a significant step forward in modeling methodology. While our procedures using two sets of coefficient estimates worked well, it should be pointed out that they indicate the need for a more generalized model, not a definitive solution to the problem.

Another possible refinement of the model is to redefine the basal area removal functions or the algorithm used to thin the stands. In most light to moderate thinnings no trees are removed from the larger diameter classes with the algorithm. However, in practice, larger trees are sometimes removed due to mortality, defect, etc. Also, this model is restricted to describing thinnings according to the removal patterns observed in the sample plots. Once data from stands thinned by other methods and diameter limit criteria become available, additional removal patterns could be formulated to simulate the various types of thinning, and thus increase the applicability and scope of this model. One method to obtain more realistic removal patterns for thinning, suggested by Cao and others (1982), is to establish stochastic models in which trees in each diameter class are assigned probabilities of being removed, and are cut or left depending on values of the random numbers generated.

Summary

In this study a growth and yield model for thinned stands of yellow-poplar was developed. The model produces both stand-level and diameter distribution level estimates of number of trees, basal area, and cubic-foot volume per acre.

Development of the model consisted of two stages. In the first, equations to predict stand-level attributes were obtained. Then, in the second, stand tables were derived from the stand-level attributes by solving for the parameters of a three parameter Weibull distribution. The shape and scale parameters were obtained according to the parameter recovery procedure. The location parameter was estimated independently. When applying the system, the same stand-level basal area equation is used when deriving diameter distributions as when estimating overall stand basal area in order to ensure compatibility between the two levels of stand detail.

Overall, the parameter recovery procedure for estimating the parameters of the diameter distributions of the stands before thinnings gave reasonable estimates of number of trees, basal area, and cubic-foot volume per acre by diameter class. The thinning algorithm, which removed a proportion of basal area from each class to simulate a thinning from below, produced stand and stock tables after thinning that were consistent with those generated before thinning, while adequately describing the observed diameter distributions after thinning. The growth and yield model for yellow-poplar provides detailed information about stand structure in an efficient manner that allows the evaluation of various thinning options.

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