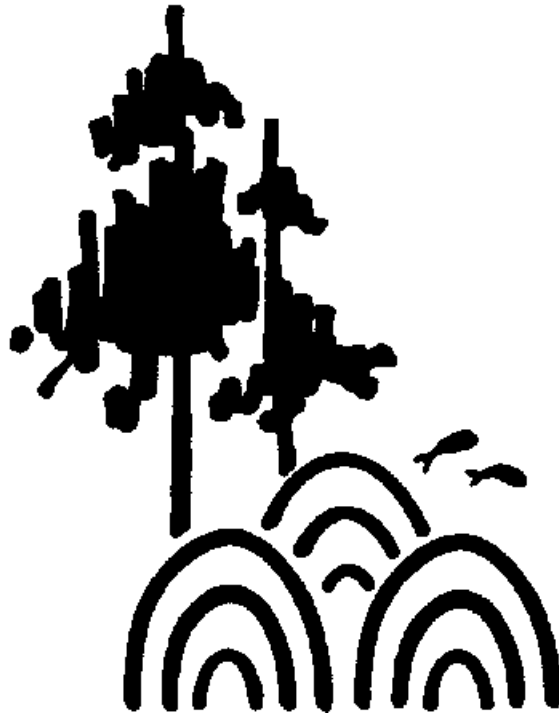


Diameter Distributions and Yields of Thinned Loblolly Pine Plantations



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School of Forestry and Wildlife Resources
Virginia Polytechnic Institute and State University
Blacksburg, Virginia 24061
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**DIAMETER DISTRIBUTIONS AND YIELDS
OF THINNED LOBLOLLY PINE PLANTATIONS**

by

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Subsequent to the original Fortran PCWTHIN published in 1982, Version 1.0 in Basic by Dr. Thomas Burk and Version 2.0 in C by Dr. Robert Weih, Dr. John Scrivani and Dr. Harold Burkhart were developed. Enhancements from these later two versions have been incorporated into version 2.1; the contributions of Drs. Burk, Weih and Scrivani are gratefully acknowledged.

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ABSTRACT

A growth and yield model for thinned loblolly pine plantations was developed using data from 128 0.2-acre permanent plots in the Virginia Piedmont and Coastal Plain. The Weibull function, used to characterize stand diameter distributions, was searched to insure that the resulting total basal area and average dbh estimates were identical to those predicted from stand variables using regression equations. Program PCWTHIN Version 2.1 is a Windows application based on Weih, et al. 1990. It allows the user to predict the growth and yield of old-field loblolly pine plantations and do basic financial analyses based on those predictions. Options are available to initialize a plantation, initialize a thinned or unthinned plantation, thin using various thinning methods, grow a stand, set values for board feet and cords, set the log rule and set minimum harvest volumes. Using PCWTHIN, the user can grow and compare, within a short period of time, numerous thinning strategies for different stands.

Trials with different thinning intensities indicated reasonable trends, as compared with published studies.

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INTRODUCTION

Growth and yield predictions are essential to forest management planning. Reliable growth and yield models assist managers in analyzing alternative management strategies. For loblolly pine (*Pinus taeda* L.), a myriad of yield information for unmanaged stands has accumulated over the years. On the other hand, yield models for thinned loblolly pine plantations still seem inadequate, and flexible models that supply information about diameter distributions are needed.

Different probability density functions (pdf's) have been used to characterize diameter distributions; most recently the beta, Weibull, and Johnson's S B distributions have been employed to develop yield estimates. The so-called probability density function approach to yield modeling involves predicting the pdf parameters from stand variables (age, site, and density) using regression techniques, and then calculating the number of trees and yield per acre in each dbh class. The drawback of this approach is that the regression models for predicting the pdf parameters usually account for only a small percentage of the variation (i.e. low R^2 values). Recently, research has been conducted to develop methods for approximating the parameters in a theoretical diameter distribution (e.g. the beta or Weibull) from overall stand values such as total basal area and mean diameter (Hyink 1980, Frazier 1981, Matney and Sullivan 1982).

The objectives of this study were: (1) to develop a whole stand model for thinned loblolly pine plantations using regression techniques, and (2) to derive diameter distributions from the predicted stand attributes by assuming that the underlying dbh distribution is Weibull distributed.

PREVIOUS WORK

Whole Stand and Diameter Distribution Models

MacKinney and Chaiken (1939) used multiple linear regression techniques to predict the logarithm of yield as a function of stand variables (age, site, density, and composition). This approach, with certain modifications, has been employed in more recent models for loblolly pine

(such as Schumacher and Coile 1960, Coile and Schumacher 1964, Goebel and Warner 1969, Burkhart et al. 1972a, 1972b) .

Growth and yield are not two separate attributes but are closely related to one another. Buckman (1962) developed a yield model for red pine where yield is obtained by mathematically integrating the growth equation over time. Clutter (1963) discussed this concept in detail and introduced a compatible growth and yield model which was later refined by Sullivan and Clutter (1972). A similar approach has been used by several other researchers including Brender and Clutter (1970), Bennett (1970), Beck and Della-Bianca (1972), Sullivan and Williston (1977), Murphy and Sternitzke (1979), and Murphy and Beltz (1981).

Diameter distributions in even-aged stands have been modeled with various probability density functions, among them the GramCharlier series (Meyer 1928, 1930; Schumacher 1928, 1930; Schnur 1934), the modified Pearl-Reed growth curve (Osborne and Schumacher 1935, Nelson 1964), Pearsonnian curves (Schnur 1934), and the log-normal distribution (Bliss and Reinker 1964).

Bennett and Clutter (1968) developed a yield model to predict multiple-product yields for slash pine plantations by using the stand table generated from a beta pdf via the Clutter and Bennett (1965) diameter distribution model. In this yield model, the parameters of the beta function that approximated the diameter distribution were predicted from stand variables (age, site, and density). The number of trees and volume per acre in each diameter class were calculated and per acre yield estimates were obtained by summing over diameter classes of interest. A similar approach was applied to loblolly pine plantations by Lenhart and Clutter (1971), Lenhart (1972), and Burkhart and Strub (1974).

The main drawback of using the beta distribution is that its cumulative distribution function (cdf) does not exist in closed form. As a result, the proportion of trees in each diameter class has to be solved by numerical integration techniques. Bailey and Dell (1973) pointed out that the Weibull distribution fits diameter data well and its cdf exists in closed form. The Weibull function was applied in plantation yield models for loblolly pine (Smalley and Bailey 1974a, Feduccia et al. 1979), slash pine (Clutter and Belcher 1978, Dell et al. 1979), and shortleaf pine (Smalley and Bailey 1974b).

Strub and Burkhart (1975) presented a class-interval-free method for predicting whole stand yield per unit area from diameter distribution models:

$$TV = N \int_L^U g(D) f(D) dD$$

where

TV	=	expected stand volume per unit area,
N	=	number of trees per unit area,
D	=	diameter at breast height,
g(D)	=	individual tree volume equation,
f(D)	=	pdf for D, and
(L,U)	=	merchantability limits for the product described by g(D).

Using this relationship, Hyink (1980) introduced a method of solving for the parameters of the pdf approximating the diameter distribution, using attributes predicted from a whole stand model. The same concept was employed by Matney and Sullivan (1982) in their model for loblolly pine plantations. In the first phase of Matney and Sullivan's study, stand volume and basal area were predicted using compatible growth and yield equations. The second phase involved solving for two parameters of the Weibull pdf which characterized the diameter distribution such that the resulting stand volume and basal area per acre would be identical to those predicted in the first phase. Frazier (1981) investigated alternative formulations for estimating parameter values in the beta and Weibull distributions from stand attributes.

Modeling Thinned Loblolly Pine Stands

Coile and Schumacher (1964) included amount of thinning as input in their model. Different types of thinning (thinning by rows, from below, or by a combination of both) can be specified in Daniels and Burkhart's (1975) and Daniels et al.'s (1979) individual tree models. Other models based on data from thinned loblolly pine stands include Clutter (1963), Brender and Clutter (1970), Sullivan and Clutter (1972), and Sullivan and Williston (1977).

The Weibull function was used by Bailey et al. (1981) to describe diameter distribution of slash pine plantations before and after thinning. Matney and Sullivan (1982) also used the Weibull distribution to produce compatible stand and stock tables for thinned loblolly pine plantations. In addition to the models mentioned above, growth and yield of thinned loblolly pine stands have been reported by many researchers (such as Bassett 1966, Bruner and Goebel 1968, Andrulot et al. 1972, Shepard 1974, Goebel et al. 1974, Feduccia and Mann 1976, Burton 1980).

DEVELOPING THE THINNED-STAND MODEL

Data

The growth and yield model for thinned loblolly pine plantations developed in this study was based on data from the Virginia Division of Forestry (VDF). This data set consists of 128 0.2-acre permanent plots from old-field plantations in the Virginia Piedmont and Coastal Plain. Number of remeasurements varied from plot to plot, ranging from 1 to 7. There were a total of 490 plot measurements.

Diameter at breast height (dbh) was recorded to the nearest inch and total height was measured to the nearest foot. Trees in the 1 and 2-inch classes were not tallied separately but combined to form one class whose midpoint was arbitrarily set at 1.5 inches. In each plot, measurements of dbh of all trees were taken but only some tree heights were measured. Height corresponding to each dbh class was predicted for each plot measurement using a regression equation of the form

$$\log_e(H) = b_0 + b_1/D,$$

where H = total tree height in feet,
 D = diameter at breast height in inches,
 b₀,b₁ = regression coefficients.

Site index was determined from the average height of the dominants and codominants in each plot, using a site index equation developed by Devan (1979).

Volumes computed by dbh class include total cubic-foot volume outside bark per acre, cordwood volume outside bark to a 4-inch outside bark top and cordwood outside bark to a 4-inch outside bark top above sawtimber using Burkhart et al.'s (1972b) individual tree volume equations and cordwood conversions; International ¼-inch, Doyle and Scribner board-foot volume to a 6-inch top diameter (ib) use the equations found in Burkhart et al (1987). Sawtimber proportions by dbh class for unthinned plantations are determined using the method of Strub (1977). For the 8-11 inch classes these proportions are 0.3246, 0.5322, 0.9385, 0.9851, respectively. For all classes greater than 11 inches the proportion is 1.0.

The stands were thinned up to 3 times and, for the most part, thinnings were from below. However, some codominants and dominants were removed to improve the quality of the leave stand. The thinnings carried out were done during routine, operational thinnings of the plantations in which the plots were located. Table 1 presents a description of plots in this data set immediately before and after thinning. The distribution of all observations by site index, age, basal area, and number of trees per acre is presented in Table 2.

Model for Thinned Loblolly Pine Plantations

The model for thinned loblolly pine plantations developed in this study consisted of two stages. In the first stage, stand-level attributes were predicted using regression techniques. The second stage involved determining the Weibull parameters so that the resulting diameter distribution would produce stand basal area and average dbh estimates identical to those predicted from regression equations in the first stage. By linking these two stages, the size-class distribution information produced is conditioned to provide aggregate values that are consistent with the predicted overall stand attributes.

Stand-Level Model

The stand-level model consisted of regression equations that predict (1) stand attributes (such as number of trees, basal area, minimum, and average diameters), and (2) density of a stand in the future (age A_2) based on stand information at present (age A_1). Also needed was a mean height equation that predicts total height corresponding to a given dbh. Table 3 shows the equations that form a whole stand model for thinned loblolly pine plantations.

Individual tree volume equations developed by Burkhart et al. (1972b) and Burkhart's (1977) volume ratio model were employed for estimating merchantable volumes. The site index equation developed by Devan (1979) was used to predict the average height of the dominants and codominants (HD) from site index and stand age, or to estimate site index from HD and stand age.

Deriving Diameter Distribution from Stand Attributes

The three-parameter Weibull pdf employed here to approximate diameter distribution is:

$$f(x) = (c/b)[(x-a)/b]^{c-1} \exp \{ -[(x-a)/b]^c \}, x \geq a,$$

where b, c = positive scale and shape parameters, respectively,
 a = nonnegative location parameter,
 x = diameter random variable.

The location parameter was predicted from a regression equation. The scale and shape parameters were searched such that the resulting Weibull distribution would produce stand basal area and arithmetic mean dbh estimates identical to those predicted from regression equations. In other words, b and c were solutions of the following system of two equations:

Table 1. Description of plots immediately before and after thinning and amount of thinning.^a

Variable	First thinning			Subsequent thinnings		
	Before	Amount	After	Before	Amount	After
<u>Number of trees/acre</u>						
Minimum	355	165	160	120	25	115
Mean	774	459	339	922	126	205
Maximum	1305	770	1040	925	435	410
<u>Basal area (sq.ft./acre)</u>						
Minimum	107	29	50	87	12	58
Mean	174	87	90	131	38	92
Maximum	227	148	145	185	77	137
<u>Total outside-bark volume (cu.ft./acre)</u>						
Minimum	1700	475	1080	2305	295	1335
Mean	3839	1910	1975	3538	944	2466
Maximum	6235	3705	3885	5935	1625	4330
<u>Average DBH (inches)</u>						
Minimum	4.5		4.0	6.0		6.3
Mean	6.4		7.1	8.9		9.2
Maximum	9.5		10.1	12.8		12.3
<u>Age (years)</u>						
Minimum	12		12	18		18
Mean	21		21	28		28
Maximum	30		30	39		39

^aDiscrepancies in the plot description (e.g., the means of a stand attribute after thinning and amount of thinning do not sum to the mean of that attribute before thinning as expected) are due to missing observations either before or after thinning.

Table 2. Distribution of all observations by site index (base age 25 years), age, basal area, and number of trees per acre.

Site Index (feet)	Age (years)	Basal Area /acre)	Number of trees per acre					Total	
			# 300	301-500	501-700	701-900	901-110		> 110
50	20	50	3	2				5	
		100	1	13				14	
		150		2	1	6		9	
		200				$\frac{1}{7}$	$\frac{2}{2}$	$\frac{3}{31}$	
				4	17	1	$\frac{1}{7}$	$\frac{2}{2}$	31
	30	50	5	2				7	
		100	33	11				44	
		150		11	2	2		15	
		200			$\frac{2}{4}$	$\frac{1}{3}$		$\frac{3}{69}$	
				38	24	4	3	69	
	40	50	1					1	
		100	22					22	
150		$\frac{5}{28}$					$\frac{5}{28}$		
			28				28		
50	100	2					2		
	150	$\frac{1}{3}$					$\frac{1}{3}$		
			$\frac{1}{3}$				$\frac{1}{3}$		
60	10	50		1				1	
		100					$\frac{1}{1}$	$\frac{1}{2}$	
					1		$\frac{1}{1}$	2	
	20	50	4	3				7	
		100	21	32				53	
		150	1	8	3	3	6	21	
200			$\frac{1}{44}$	$\frac{7}{10}$	$\frac{8}{11}$	$\frac{2}{8}$	$\frac{18}{99}$		
		26	44	10	11	8	99		

Table 2. Distribution of all observations by site index (base age 25 years), age, basal area, and number of trees per acre (continued).

Site Index (feet)	Age (years)	Basal Area /acre)	Number of trees per acre					Total	
			# 300	301-500	501-700	701-900	901-110		> 110
60	30	50	6					6	
		100	88	11				99	
		150	19	20	2			41	
		200			$\frac{1}{3}$	$\frac{1}{7}$		$\frac{2}{7}$	
			113	31				148	
	40	100	23					23	
		150	$\frac{20}{43}$					$\frac{20}{43}$	
			43					43	
	50	100	2					2	
		150	2					2	
200		$\frac{3}{7}$					$\frac{3}{7}$		
70	10	50	2	2	2			6	
		100		4	2	1		7	
		150				$\frac{4}{5}$	$\frac{4}{4}$	$\frac{2}{2}$	$\frac{10}{23}$
			2	6	4			23	
	20	100	7	11	3			21	
		150	1	6	1			8	
		200			$\frac{2}{6}$	$\frac{2}{2}$		$\frac{4}{33}$	
			8	17				33	
	30	100	1					1	
		150	$\frac{3}{4}$					$\frac{3}{4}$	
		4					4		
Total			276	140	28	29	15	2	490

Table 3. Regression equations that form a whole stand model for thinned loblolly pine plantations.

Equation Number	Equation ^a
1	$\ln(B_2) = 5.40816 + 0.0032121 S - (A_1/A_2) \quad [5.40816 + 0.0032121 S - \ln(B_1)]$
	$n = 207; \overline{\ln(B_2)} = 4.7230; s_{y,x} = 0.0860$ $R^2 = 99.34\%; R^2(B_2) = 80.47\%$
2	$N_2 = [N_1^{-0.65808} + 0.0000075795 (A_2^{1.78019} - A_1^{1.78019})]^{-1/0.65808}$
	$n = 207; \overline{N_2} = 253.02; s_{y,x} = 18.64$
	$R_2 = 97.07\%; R^2(N_2) = 97.07\%$
3	$\ln(B) = -4.39181 + 0.19054 / A + 1.34753 \ln(HD) + 0.63902 \ln(N)$
	$n = 490; \overline{\ln(B)} = 4.7149; s_{y,x} = 0.1407$
	$R^2 = 75.48\%; R^2(B) = 77.01\%$
4	$\ln(N) = 7.79805 + 2.10495 / A - 1.89908 \ln(HD) + 1.16744 \ln(B)$
	$n = 490; \overline{\ln(N)} = 5.6732; s_{y,x} = 0.1902$
	$R^2 = 87.19\%; R^2(N) = 85.78\%$
5	$\ln(H) = 0.46152 + 0.43275 / A + 0.93333 \ln(HD) = 0.08583 \ln(B)$
	$+ 0.07596 \ln(N) - 2.14312 / D$
	$n = 3559; \overline{\ln(H)} = 4.0404; s_{y,x} = 0.0422$
	$R^2 = 96.76\%; R^2(H) = 97.62\%$

Table 3. Regression equations that form a whole stand model for thinned loblolly pine plantations (continued).

Equation Number	Equation ^a
6	$\ln(D_{min}) = 1.10835 + 5.10755 / A + 0.50531 \ln(HD) + 0.28544 \ln(B) = 0.57131 \ln(N)$
	$n = 427; \overline{\ln(D_{min})} = 1.5253; s_{y,x} = 0.2972$ $R^2 = 46.84\%; R^2(D_{min}) = 51.02\%$
2	$\ln(Dq - \bar{D}) = -9.05733 + 0.89274 \ln(HD) + 0.58151 \ln(N)$
	$n = 489; \overline{\ln(Dq - \bar{D})} = -2.1316; s_{y,x} = 0.6206$ $R^2 = 11.507\%; R^2(\bar{D}) = 97.07\%$

^aNotation:

- $\ln(x)$ = Natural logarithm of x,
- $R^2(x)$ = Percent variation of x explained by the model,
- A = Stand age in years,
- B = Basal area in square feet per acre,
- D = Tree diameter at breast height (dbh) in inches,
- \bar{D} = Arithmetic mean dbh in inches,
- D_{min} = Minimum dbh in inches,
- D_q = Quadratic mean dbh in inches,
- H = Total height in feet of a tree having dbh D,
- HD = Average height in feet of the dominants and codominants,
- N = Number of surviving trees per acre,
- S = Site index in feet (base age 25 years).

Subscript i denotes that the measurement is taken at time i.

$$\hat{D} = \int_a^{\infty} xf(x)dx \quad (8)$$

$$\hat{B} = 0.005454N \int_z^{\infty} x^2 f(x)dx \quad (9)$$

where D = predicted arithmetic mean dbh in inches,
 \hat{B} = predicted basal area in square feet per acre,
 N = number of surviving trees per acre,
 $f(x)$ = Weibull pdf with parameters a , b , and c .

Equation (8) can be rewritten as

$$\hat{D} = a + b \Gamma(1 + 1/c) \quad (10)$$

or

$$b = (\hat{D} - a) / \Gamma(1 + 1/c) \quad (11)$$

where $\Gamma(x)$ = gamma function evaluated at x .

In most diameter distribution models, stand volume and basal area are often obtained by first computing these attributes for each dbh class and then summing over diameter classes of interest. Equation (9) can be approximated in a similar manner by replacing the integral sign with a summation sign:

$$B = 0.005454N \sum_{x_i=1}^{\infty} x_i^2 f_i \quad (12)$$

where x_i = midpoint of the i th dbh class,
 f_i = $F(x_i+0.5) - F(x_i-0.5)$ = proportion of trees in the i th dbh-class,
 $F(x)$ = $1 - \exp \{ -[(x-a)/b]^c \}$ = Weibull cumulative distribution function with parameters a , b , and c .

The iterative technique used to solve for the Weibull parameters is a combination bisection and false position routine. The complete gamma function is obtained by using the approximation suggested by Lanczos (1964) and Press et al. (1988). The incomplete gamma function is approximated using the method suggested by Press et al. (1988).

RESULTS AND DISCUSSION

Program PCWTHIN

All of the techniques described earlier were incorporated into program PCWTHIN. This program can generate stand and stock tables for different combinations of site, stand age, and density. It is also able to simulate a loblolly pine stand for a specified period during which thinning options are available at any point in time.

Prediction of the Present Stand

The inputs needed are:

- (1) age of the present stand,
- (2) site index (or average height of the current dominants and codominants),
- (3) two measures of density (total basal area and number of trees per acre).

If only one measure of density is available, the other can be estimated by employing the appropriate equation (3 or 4) of Table 3. Equations (6, 7) of Table 3 predict the minimum and arithmetic mean dbh of the stand. The Weibull location parameter a is computed from D_{min} as follows:

$$a = \text{FLOOR}(D_{min} - 0.5) - 0.49,$$

where $\text{FLOOR}(x) =$ integer portion of x .

This adjustment simply sets D_{min} at the lower end of its 1-inch dbh class and then decreases it by 1 inch.

The Weibull parameters b and c are obtained by solving equation (12). As a result, number of trees and basal area per acre for each dbh class can be computed. The mean height equation (equation 5 of Table 3) predicts total height corresponding to the midpoint of each dbh class. Total volumes outside and inside bark can be obtained from the individual tree volume equations published by Burkhart et al. (1972b). Merchantable volumes can also be calculated using the volume ratio methods developed by Burkhart (1977) and Cao and Burkhart (1980).

Thinning

Inputs for the thinning option include age of the stand when thinning occurs and type of thinning. Thinning can be carried out by rows, from below, or a combination of both.

It is assumed that the diameter distribution does not change due to row thinning. Thus the number of trees, basal area, and volume per acre in each dbh class are reduced by the proportion of trees removed in thinning.

Thinning from below is defined here as removing all trees with dbh values less than a specified diameter. Input for this type of thinning can be either this diameter limit or a residual basal area. A combination of row and low thinning involves first a row thinning followed by a thinning from below.

Projection

Basal area and number of trees per acre at some age in the future can be projected using equations (1) and (2) of Table 3 for thinned stands, or the following equations from Coile and Schumacher (1964) for unthinned loblolly pine plantations:

$$\log_{10}(N) = \log_{10}(N_0) + [2.1346 - 1.1103 \log_{10}(N_0) + 0.1384 (OF)] A/100$$

$$\log_{10}(B) = 1.4366 \log_{10}(S) - 0.7084 (10/A) + 0.4888 \log_{10}(N) + 0.0585 (OF) - 1.4436$$

where

- A = age in years,
- B = stand basal area in square feet per acre at age A,
- N = number of surviving trees per acre at age A,
- N₀ = number of trees planted per acre,
- OF = +1 if old-field origin, and -1 otherwise,
- S = site index in feet (base age 25 years).

Procedures similar to those for predicting the present stand are then employed to produce stand and stock tables for the future stand.

Diameter Distribution of a
Previously Low-Thinned Stand

Suppose that in a previous thinning from below, all trees having dbh below D_{thin} were cut. If the predicted Weibull location parameter (a) for the present stand is greater than or equal to D_{thin} , then the complete Weibull function is used to characterize the current diameter distribution. On the other hand, when a is less than D_{thin} , a left-truncated Weibull pdf is more appropriate where D_{thin} is the truncation point.

When the truncated Weibull is employed, equation (10) is replaced with:

$$\hat{D} = a + \int_{(D_{thin}-a)}^{\infty} \frac{x(c/b)(x/b)^{c-1} \exp[-(x/b)^c]}{1 - F(D_{thin})} dx$$

$$\hat{D} = a + \frac{b}{1 - F(D_{thin})} \int_{\left(\frac{D_{thin}-a}{b}\right)^c}^{\infty} y^{1/c} \exp(-y) dy$$

or

$$\hat{D} = a + \frac{b}{1 - F(D_{thin})} \left[(1 + 1/c) - \int_0^{\left(\frac{D_{thin}-a}{b}\right)^c} y^{1/c} \exp(-y) dy \right] \quad (13)$$

where $F(x) = 1 - \exp \{ -[(x-a)/b]^c \}$.

The procedures for deriving the parameters of the truncated Weibull pdf are similar to those of the complete Weibull described earlier. The shape parameter c is solved from equation (12); for each estimated value of c , the scale parameter b is obtained from equation (13) (instead of from equation (11) as in the case of the complete Weibull pdf). The proportion of trees in the i th dbh class of the truncated distribution is given by:

$$f_i = \frac{F(i + 0.5) - F(i - 0.5)}{1 - F(D_{thin})}$$

Effect of Thinning Regimes on Yield

In order to demonstrate the effect of thinning type and intensities on yield, different thinning options were applied to loblolly pine plantations on site index 60 soil. These hypothetical stands had 800 trees and 130 sq.ft. per acre of basal area at age 15, and would be harvested at age 30. Option D was the control where no thinning was applied. In the rest of the thinning options, the stands were thinned repeatedly at ages 15, 20, and 25 to a specified residual basal area. Residual basal areas were arbitrarily set at 80, 95, and 110 sq.ft. per acre for options A, B and C, respectively. Three types of thinning were considered for each residual density: (1) row thinning, (2) low thinning, and (3) a combination of row and low thinnings, where 25% of the basal area removed was first cut in a row thinning and then the remainder from a thinning from below. Option B1, for example, means row thinning to 95 sq.ft./acre of residual basal area.

Yields of these stands under different regimes are presented in Table 4. Total cubic-foot volume production (amount removed in thinnings plus final harvest volume) did not differ much from row to low thinning for a given thinning level. Note that thinning level is to a specified residual basal area and that number of trees remaining therefore varies by thinning type. Stand average diameter, however, was lowest in row thinning, highest in low thinning, and somewhere between these two extremes in the combination of row and low thinnings, as expected. As found by other researchers (such as Feduccia and Mann 1976, Sullivan and Williston 1977), cubic-foot volume production increased with higher residual basal area. On the other hand, average dbh increased as the thinnings were more severe, which implies an increase in board-foot volume production.

A fourth thinning option, called "Thinomatic" removes trees according to the average pattern observed in many operational thinnings. The proportion of basal area removed in a 1-inch dbh class according the thinomatic rule is given by the following equation (Burk et al. 1984).

$$P_i = \exp[-0.73148 (D_i^2/Q^2)^{1.45759}]$$

where: P_i = proportion of basal area to remove in class I
 D_i = midpoint dbh of class I
 Q = quadratic mean dbh before thinning.

Basal area is removed according to the equation starting in the smallest dbh class and working upward until the desired residual basal area remains. If the entire dbh distribution is gone through without removing the required basal area, the remainder is obtained by removing all trees in the smallest dbh classes until the specified residual basal area is reached. Whenever only a proportion of the trees in a dbh class are removed, the remaining trees are assumed to be uniformly distributed across the diameter class.

Table 4. Total cubic-foot yield on a per acre basis of a loblolly pine plantation on site 60 land, with 800 trees and 130 square feet of basal area at age 15, by thinning option.

Age (years)	Before thinning				After thinning				Total Volume Production (cu. ft.)	
	Number of trees	Basal Area (sq. ft.)	Average DBH (inches)	Total Volume ob (cu. ft.)	Number of trees	Basal Area (sq. ft.)	Average DBH (inches)	Total Volume ob (cu. ft.)		Volume removed (cu. ft.)
<u>OPTION A1: Row thinning – Residual basal area = 80 sq. ft./acre</u>										
15	800	130	5.3	2225	492	80	5.3	1369	856	2225
20	466	108	6.4	2375	343	80	6.4	1751	624	3231
25	326	102	7.4	2643	255	80	7.4	2071	572	4123
30	242	98	8.5	2860						4912
<u>OPTION A2: Low thinning – Residual basal area = 80 sq. ft./acre</u>										
15	800	130	5.3	2225	350	80	6.4	1381	844	2225
20	335	108	7.06	2375	209	80	8.3	1771	604	3219
25	202	102	9.5	2652	139	80	10.2	2097	555	4100
30	134	98	11.5	2868						4871
<u>OPTION A3: 25% row thinning and 75% low thinning – Residual basal area = 80 sq. ft./acre</u>										
15	800	130	5.3	2225	367	80	6.3	1378	847	2225
20	351	108	7.4	2376	221	80	8.1	1770	606	3223
25	212	102	9.3	2652	149	80	9.9	2094	558	4105
30	143	98	11.1	2868						4879

Table 4. Total cubic-foot yield on a per acre basis of a loblolly pine plantation on site 60 land, with 800 trees and 130 square feet of basal area at age 15, by thinning option (continued).

Age (years)	Before thinning				After thinning				Total Volume Production (cu. ft.)	
	Number of trees	Basal Area (sq. ft.)	Average DBH (inches)	Total Volume ob (cu. ft.)	Number of trees	Basal Area (sq. ft.)	Average DBH (inches)	Total Volume ob (cu. ft.)		Volume removed (cu. ft.)
<u>OPTION B1: Row thinning – Residual basal area = 95 sq. ft./acre</u>										
15	800	130	5.3	2225	585	95	5.3	1625	590	2225
20	550	123	6.3	2700	423	95	6.3	2078	622	3290
25	398	117	7.2	3028	323	95	7.2	2456	572	4240
30	304	113	8.1	3294						5078
<u>OPTION B2: Low thinning – Residual basal area = 95 sq. ft./acre</u>										
15	800	130	5.3	2225	454	95	6.2	1633	592	2225
20	430	123	7.1	2700	274	95	7.9	2104	596	3292
25	261	117	9.0	3038	188	95	9.6	2485	553	4226
30	180	113	10.6	3305						5046
<u>OPTION B3: 25% row thinning and 75% low thinning – Residual basal area = 95 sq. ft./acre</u>										
15	800	130	5.3	2225	470	95	6.0	1631	594	2225
20	446	123	7.0	2699	293	95	7.6	2098	601	3293
25	279	117	8.6	3037	201	95	9.2	2483	554	4232
30	192	113	10.3	3305						5054

Table 4. Total cubic-foot yield on a per acre basis of a loblolly pine plantation on site 60 land, with 800 trees and 130 square feet of basal area at age 15, by thinning option (continued).

Age (years)	Before thinning				After thinning				Total Vol. Production (cu. ft.)	
	Number of trees	Basal Area (sq. ft.)	Average DBH (inches)	Total Volume ob (cu. ft.)	Number of trees	Basal Area (sq. ft.)	Average DBH (inches)	Total Volume ob (cu. ft.)		Volume removed (cu. ft.)
<u>OPTION C1: Row thinning – Residual basal area = 110 sq. ft./acre</u>										
15	800	130	5.3	2225	677	110	5.3	1883	342	2225
20	632	138	6.2	3013	504	110	6.2	2406	607	3355
25	472	132	7.0	3401	394	110	7.0	2841	560	4350
30	368	128	7.8	3717						5226
<u>OPTION C2: Low thinning – Residual basal area = 110 sq. ft./acre</u>										
15	800	130	5.3	2225	564	110	5.9	1885	340	2225
20	531	138	6.8	3010	357	110	7.4	2430	580	3350
25	338	132	8.3	3410	246	110	9.0	2875	535	4330
30	234	128	9.9	3730						5185
<u>OPTION C3: 25% row thinning and 75% low thinning – Residual basal area = 110 sq. ft./acre</u>										
15	800	130	5.3	2225	573	110	5.9	1884	341	2225
20	539	138	6.7	3010	372	110	7.3	2425	585	3351
25	352	132	8.2	3409	264	110	8.6	2869	540	4335
30	250	128	9.6	3728						5194

Table 4. Total cubic-foot yield on a per acre basis of a loblolly pine plantation on site 60 land, with 800 trees and 130 square feet of basal area at age 15, by thinning option (continued).

Age (years)	Before thinning				After thinning				Total Volume Production (cu. ft.)	
	Number of trees	Basal Area (sq. ft.)	Average DBH (inches)	Total Volume ob (cu. ft.)	Number of trees	Basal Area (sq. ft.)	Average DBH (inches)	Total Volume ob (cu. ft.)		Volume removed (cu. ft.)
<u>OPTION D: No thinning</u>										
15	800	130	5.3	2225						2225
30	540	186	7.8	5387						5387

Table 5. Comparison of predicted yields of Coile and Schumacher (1964) and those from program WITHIN on a per acre basis for thinned loblolly pine plantations.

Source	Site Index (feet)	Number of trees at age 5	Basal Area (sq. ft.) at age 5	Age when thinned (years)	Amount of thinning		Residual stand at age 30			Total Volume Production (cords)	
					Basal area (sq. ft.)	Volume ^b (cords)	Quadratic mean DBH (inches)	Number of trees	Basal area (sq. ft.)		Volume (cords)
C&S ^c	50	600	9.9	20	68	10	13.3 (7.7) ^f 9.0	140 (365) 172	135 (118) 76	28.7 (25.7) 19.6	38.7 (26.7) 29.6
Row ^d Low ^e					58 61		10.8	114	72	18.7	28.7
C&S	50	800	11.4	20	82	12	13.4 (7.3)	146 (448)	142 (130)	30.3 (29.1)	42.3 (29.1)
Row Low					72 77		8.6 10.8	184 106	74 68	18.9 17.5	30.8 29.5
C&S	60	600	12.9	17,22	45,36	7,7	13.6 (8.8)	168 (365)	170 (153)	43.7 (42.9)	47.7 (42.9)
Row Low					38,29 43,30		9.7 12.1	202 122	104 97	31.2 29.4	45.0 43.3
C&S	60	800	14.8	17,22	58,47	9,9	14.6 (8.3)	159 (448)	185 (169)	47.1 (47.2)	65.1 (47.2)
Row Low					51,38 59,38		9.2 12.3	207 105	96 87	28.8 26.3	46.7 44.3
C&S	70	600	16.1	15,20,25	37,37,39	6,8,10	15.1 (9.8)	158 (365)	196 (191)	60.6 (63.4)	84.6 (63.4)
Row Low					31,31,33 36,31,33		10.4 13.6	178 99	104 100	35.9 34.3	60.0 58.0
C&S	70	800	18.5	15,20,25	43,47,51	7,8,13	14.7 (9.3)	189 (448)	222 (211)	68.2 (70.0)	92.2 (70.0)
Row Low					37,39,43 46,40,43		9.7 13.7	189 85	97 87	32.2 30.0	63.1 59.9

^aSite index at base age 25 years; ^bCord volume to a 4-inch top, converted from cubic-foot volume outside bark to a 4-inch top, using ratios from Burkhardt *et al.* (1972b); ^cCoile and Schumacher (1964); ^dRow thinning, program WITHIN; ^eLow thinning, program WITHIN; ^fNumbers in parentheses are for unthinned stands

Table 6. Comparison of observed yields of Goebel et al. (1974) and predicted yields from program WTHIN on a per acre basis for thinned loblolly pine plantations.

Source	Site Index (feet)	Age (years)	Number of trees	Basal Area (sq. ft.)	Average DBH (inches)	Total Volume (cu. ft.)	Age (years)	Number of Trees	Basal Area (sq. ft.)	Average DBH (inches)	Residual Volume (cu. ft.)	Age when thinned (years)	Basal area limit (sq. ft.)	Volume removed in thinning (cu. ft.)	Total Volume Production (cu. ft.)					
Observed Row ^a Low ^d	51a	13	790	131	5.3	1476	34	140	75	9.9	1870	13,21	75	2325	4195					
	60b				5.2					1491						9.8	1967	27,34	2644	4611
	60b				5.2					1491						14.2	1971	2547	4519	
Observed Row Low	51	13	800	116	5.0	2116	34	160	84	9.8	2075	13,21	85	2188	4263					
	60				5.0					1422						9.2	2224	27,34	2456	4680
	60				5.0					1422						13.2	2240	2345	4585	
Observed Row Low	51	13	780	129	5.3	1579	34	160	94	10.4	2349	13,21,	95	2189	4538					
	60				5.4					1600						9.4	2485	27,34	2488	4973
	60				5.4					1600						13.1	2502	2374	4876	
Observed Row Low	51	13	1016	124	4.6	1409	34	132	80	10.5	2065	13,18,20	80	2261	4326					
	60				4.6					1494						8.8	2089	25,34	2536	4625
	60				4.6					1494						13.5	2110	2419	4529	
Observed Row Low	51	13	1004	122	4.6	1350	34	148	89	10.5	2436	13,18,20	90	2431	4867					
	60				4.6					1469						8.4	2345	25,34	2388	4733
	60				4.6					1469						12.8	2376	2258	4635	
Observed Row Low	51	13	924	105	4.5	1133	34	176	103	10.4	2934	13,18,20,	100	2707	5641					
	60				4.4					1254						7.9	2595	25,34	2034	4629
	60				4.4					1254						11.4	2647	1896	4542	
Observed Row Low	55	17	1180	196	5.3	1133	30	252	85	7.8	2107	17,20,	85	2401	4508					
	61				5.3					1254						7.9	2106	24,30	3034	5140
	61				5.3					1254						12.2	2142	2894	5036	
Observed Row Low	55	17	1220	187	5.4	3054	30	280	111	8.6	2854	17,20,	110	2192	5046					
	61				5.1					3000						7.2	2704	24,30	2446	5151
	61				5.1					3000						10.5	2771	2280	5051	
Observed Row Low	55	17	1212	180	5.3	2884	30	372	129	8.0	3232	17,20,	135	1896	5128					
	61				5.0					2880						6.8	3302	24,30	1842	5144
	61				5.0					2880						9.4	3391	1658	5048	

^aSite index (base age 25 years) from Goebel and Shipman (1964).^bSite index (base age 25 years) from Clutter and Lenhart (1968).^cRow thinning, program WTHIN^dLow thinning, program WTHIN

Although only total cubic-foot volume is presented in Table 4, users can readily develop yield tables in other units (cords, board feet, pounds, etc.) and for any specified portion of the stand by substituting appropriate volume or weight equations and specifying desired threshold diameters in the model.

Comparison with Published Information on Thinning

Coile and Schumacher's (1964) Model

Program PCWTHIN was compared with the model for thinned loblolly pine plantations developed by Coile and Schumacher (1964); results are presented in Table 5. Both row and low thinning options were tried, for the thinning in practice would likely be somewhere between these two cases. Care was taken such that cord volume removed in each thinning was identical to that specified by Coile and Schumacher. Examination of the residual stands at age 30 revealed that the number of surviving trees from Coile and Schumacher's model was between the predicted values from the two types of thinning of program PCWTHIN. Residual basal area, quadratic mean dbh, and volume from Coile and Schumacher's predictions were consistently higher than those from PCWTHIN.

Coile and Schumacher's predicted total volume production of thinned stands far exceeded that of unthinned counterparts. On the other hand, total volume predictions (i.e., volume removed in thinnings plus residual volume) of thinned stands at age 30 from program PCWTHIN were close to volumes of unthinned stands at age 30 from Coile and Schumacher's model. This agrees well with what other investigators have found, namely, that total cubic-foot volume production is generally little affected by thinning (Smith 1962, Andrulot et al. 1972, Goebel et al. 1974).

Yields Reported by Goebel et al. (1974)

Goebel et al. (1974) reported yields of 9 old-field loblolly pine stands; each had been thinned 4 to 5 times to a specified residual basal area per acre. Site indices were determined from curves developed by Goebel and Shipman (1964). Goebel and Warner (1969) recognized a significant site-age bias in these site index curves and revised their yield model using Clutter and Lenhart's (1968) polymorphic site index curves. Devan's (1979) site index equation was replaced with that of Clutter and Lenhart (1968) in program WTHIN when simulating the stands based on the guidelines set forth by Goebel et al. (1974). Data for total cubic-foot volumes reported by Goebel et al. (1974) were based on volume tables prepared by MacKinney and Chaiken (1939). Thus MacKinney and Chaiken's (1939) individual tree volume equation was used in this simulation.

The observed number of trees per acre and average dbh in each plot fell between values

predicted from PCWTHIN using the row and low thinning options (Table 6). Comparison of total volume production in these 9 stands shows that the mean relative difference between observed and predicted yields (averages of yields from the row and low thinning options) is -2.52%.

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PCWTHIN 2.1 USER'S MANUAL

Preface

The equations that comprise PCWTHIN 2.1 have been programmed into a Windows application for implementation with Windows 95, 98, NT or 2000 operating systems. The PCWTHIN 2.1 software is available for \$50 by contacting:

Department of Forestry
Virginia Tech
Blacksburg, VA 24061

Most of the functionality of the PCWTHIN 2.1 software follows that of other Windows applications and experienced users of Windows software should have no trouble implementing PCWTHIN 2.1. There are, however, nuances peculiar to this application for which additional explanation may be helpful.

Purpose of PCWTHIN 2.1

PCWTHIN 2.1 is a computer program which can be used to predict the growth and yield of thinned or unthinned old field loblolly pine plantations and do basic financial analyses based on those predictions. Predictions are obtained by choosing options from pop-up menus and responding to requests for stand level characteristics on a per acre basis. Results are displayed in terms of trees per acre, basal area and various volumes per acre by one-inch diameter at breast height (dbh) classes. The diameter distribution of the stand can be displayed as a 3D bar graph. At the end of a session, a stand summary and financial analysis of that stand summary can be displayed. If a parallel printer is attached to the computer system, all output on the screen can be printed. Options are available to initialize a plantation, initialize a thinned or unthinned plantation, thin a plantation using various methods, grow a stand, set values for board feet and cords, set the log rule and set minimum harvest volumes. Using PCWTHIN 2.1, the user can grow and compare, within a short period of time, numerous thinning strategies for different stands.

Initializing a plantation

There are three initialization options presented on the toolbar: initializing an existing unthinned, existing thinned or new plantation. For each initialization option, a dialog box appears to accept necessary input from the user. For unthinned stands the basal area is optional input. An existing stand between the ages of 10 and 50 can be initialized with just the site index, age, number of trees surviving and/or basal area.

When the INITIALIZE EXISTING THINNED PLANTATION option is selected from the Initialize menu PCWTHIN 2.1 prompts for the current age of the stand, the site index, the basal area and/or number of trees and the smallest diameter class that contains trees at the initialization age. The diameter limit must be greater than or equal to zero and at least one dbh class below average dbh. If the diameter limit is unknown, insert a 0 (zero).

The third option of the Initialize menu is to INITIALIZE A NEW PLANTATION. When this option is chosen PCWTHIN 2.1 prompts for the number of trees planted and the site index of the stand.

Output options

The PCWTHIN 2.1 stand table output displays four columns of volumes. The first and second volume columns display total outside bark and total cord volumes, respectively. The third and fourth columns (Pulpwood + Sawtimber Volume) present the stand assuming it has been merchandized into two mutually exclusive products: pulpwood and sawtimber. Topwood from sawtimber trees is included in the pulpwood portion of the volume.

The Options main menu item allows the user to select between Doyle, Scribner or International $\frac{1}{4}$ board-foot rules.

Copying output

Stand and stock table output values can be highlighted with the arrow keys or by dragging the mouse and then copied to the Windows clipboard. From the clipboard they can be pasted into other Windows applications such as spreadsheets or graphics packages. This facilitates further analyses of PCWTHIN 2.1 simulation results.

Program initialization limits and error messages

The stand age must be less than 51 years old. Site index can be between 40 and 80 based on a base age of 25. Basal area must be greater than 60 and less than 250. Trees per acre must be greater than 125 and less than 1350. If data outside these limits are specified, an error message will appear. If an unrealistic combination of inputs is specified projections and predictions will be unrealistic.

Thinomatic thinning

The THINOMATIC thinning option will prompt for the basal area desired after thinning. The desired basal area must be greater than 50 square feet per acre and less than the current basal area. The THINOMATIC method removes trees according to the average pattern observed in certain types of operational thinnings where all diameter classes are subject to removals. The proportion of basal area removed in a 1-inch dbh class according to the thinomatic rule is given by the following equation (Burk et al., 1984):

$$P_i = \exp[-.73148*(D_i^2/Q^2)^{1.45759}]$$

where: P_i = proportion of basal area to remove in class i

D_i = midpoint dbh of class i

Q = quadratic mean dbh before thinning

Basal area is removed according to the equation starting in the smallest dbh class and working upward until the desired residual basal area remains. If the entire dbh distribution is gone through without removing the required basal area, the remainder is obtained by removing all trees in the smallest dbh classes until the specified residual basal area is reached. Whenever only a portion of the trees in a dbh class are removed, the remaining trees are assumed to be uniformly distributed across the diameter class.

Row thinning

The ROW option will prompt for the desired residual basal area after thinning. The desired basal area must be greater than 50 square feet per acre and less than the current basal area. The ROW option removes a constant proportion from each dbh class. The proportion is equal to $1.0 - (\text{basal area after thinning} / \text{basal area before thinning})$.

Low thinning

There are two LOW thinning limit options: a DBH limit and a residual basal area limit. The SPECIFY DBH option allows specification of the threshold dbh. When specifying a threshold dbh, all trees below the threshold dbh will be removed. Specifying a threshold dbh that will leave less than 50 square feet of basal area causes the program not to remove all the trees below the threshold dbh. The program will leave at least 50 square feet of basal area. The SPECIFY BASAL AREA option prompts for a residual basal area in square feet after a low thinning. The basal area specified must be between 50 square feet and the current basal area shown on the status line. Trees will be removed starting at the smallest diameter class until the remaining basal area is what was specified.

Row/Low thinning

The ROW/LOW thinning option will prompt for the desired residual basal area after thinning and the percent of basal area to remove by row thinning. Basal area after thinning must be between 50 square feet and the current basal area shown on the status line. Percent basal area removed by row thinning must be between 1 and 100 percent.

Under the ROW/LOW thinning option, the basal area to be row thinned is removed first. This is done by computing basal area before and after row thinning and removing a constant proportion of trees from each dbh class as discussed under Row Thinning. The remainder of the basal area to be removed is obtained from the smallest dbh classes as discussed under Low Thinning.

The ROW/LOW thinning option is used to obtain thinning results from below with a different stand structure than that provided by the THINOMATIC option. The ROW/LOW option differs from conducting a row followed by a low thinning in two respects. In the ROW/LOW option only one stock table is displayed. Using a row and then a low thinning two different stock tables will be displayed. Also, no provision is made to specify low thinning in terms of a threshold dbh class under the ROW/LOW option.

Growing a plantation

To grow a plantation, click the Grow toolbar button and specify a future age. A plantation must be initialized before it can be grown. The projected age must be greater than the current age and less than 50 years.

Harvesting a plantation

To harvest a plantation, click the HARVEST option from the toolbar. A stand history of management activity since initialization is presented including the age, action performed on the plantation, trees per acre, basal area, cubic foot volume, cords and board foot volume based on the log rule in effect when the action was performed. At the bottom of the table, the total volume removed is shown. In the Cords column only one product (pulpwood) is assumed of interest, whereas in the Pulpwood+Sawtimber column integrated utilization for the two products is assumed. The minus signs indicate removals.

Financial analysis

When the INCLUDE FINANCIAL ANALYSIS option (under the Options main menu item) has been checked, a financial analysis summary is displayed. The Financial Analysis summary displays any action that removed volume. It displays the age in which the action was performed, the action performed, the dollar value returned for cords, the dollar value returned for board feet, the present value for cords and the present value for board feet and top cords. Financial values (interest rate and dollar values for cordwood and sawtimber) are set by the user under the Options main menu item. If the volume removed per acre is less than the minimum commercial volumes, the dollar value is zero. The present value sum for cords and board feet and top cords are given based on separate products. The present value is used to determine the value today of some future value. The equation used for present value in PCWTHIN 2.1 is shown below.

$$V_0 = V_n / (1 + i)^n$$

where: V_0 = present value

V_n = value of product in the future

n = number of periods (years) in the future

i = interest rate per period (year)

Log rules

This option allows selecting the log rule used to determine board feet. The default is International 1/4 Log Rule; Doyle or Scribner can also be chosen. The Stand History will reflect all changes in log rules used during the projection. **Warning:** It is possible to switch log rules at any point in the simulation to view stand tables generated from different log rules. However, this is not recommended if a harvest is anticipated followed by a financial analysis because it may result in multiple log rules being used to calculate board feet in the Stand History.

Financial analysis values

This option allows changing the interest rate, product values and minimum commercial volumes to be considered commercial products. These values are used to compute a financial analysis of the stand. When the Financial analysis option is checked, PCWTHIN 2.1 displays the results of the financial analysis at harvest.

Diameter bar graph

Placing a check mark beside this option displays a bar graph of the diameter distribution below the stand table.